Part 1: Understanding the components of the fundamental sampling error: a key to good sampling practice

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Synopsis

The variety and sources of sampling errors have been studied since the late 1800s, but the pioneering work of Pierre Gy in the 1950s provided insights into sampling as an error generating process. Total sampling variance is a measure of the total sampling error, but disaggregating the variance into its component parts is problematical. Ten sources of error and error generating processes have been identified, some of which can be eliminated through rigorous application of the principle of correct sampling. Others can only be minimized but never eliminated. Three principal questions relating to the appropriate mass of a sample, the fragment size of the sample and the size of the associated error are addressed. These questions relate principally to the Fundamental Error (FE), the only error that can be estimated before the sampling event. Aspects of constitutional and distributional heterogeneity are examined, as are the essential components of Gy's formula including the shape factor (f), the granulometric factor (g), the mineralogical composition factor (c), and the liberation factor, ℓ. Applications of Gy's equation to the problems of sampling in the minerals industry are explained using real-life examples. The approach to sampling taken by Pierre Gy provides an excellent inferential relationship between the mass of material to be sampled, the size of the fragments being sampled, and the relative variance of the sampling error. It also provides solutions for three of the main problems associated with the Theory of Sampling.

Pierre Gy's sampling errors

The current emphasis on understanding the variety and source of sampling errors has arisen out of the pioneering work of Pierre Gy who in 1951 wrote an unpublished paper in French entitled 'Minimum mass of a sample needed to represent a mineral lot'. This was the first in a series of publications translated into English that established Gy as the leading authority on issues of sampling broken ores, although Brunton (1895) had earlier explored some of the problems associated with this activity. Gy (1973, 1979, 1982, 1992, 1995, and 1998) identified the action of sampling as an error-generating process consisting of seven principal errors. The number of errors has grown over time and now stands at ten, all of which Gy identified and were implicit in his analysis of the errors, although he had not explicitly named them. The development and progression of understanding these errors has been highlighted by Pitard (1993). Contrary to the popular belief that errors are self-compensating, sampling variances are additive. The simplest way of disaggregating the overall sampling variance, is to separate it into the component parts that arise at each stage of the process. Listed below are the ten sources of sampling error (Pitard, 2005) that contribute to the non-representativeness of samples. They include:

- *In situ* Nugget Effect (NE)
- Fundamental sampling error (FE)
- Grouping and segregation errors (GE)
- Long-range heterogeneity (quality) fluctuation error (shifts and trends, QE1)
- Long-range periodic heterogeneity (quality) fluctuation error (cycles, QE2)
- Increment delimitation error (DE)
- Incremental extraction error (EE)
- Weighing error (WE)
- Preparation error (PE)
- Analytical error (AE).

The total sampling error (TE) can be split into separate components, as shown by Gy (1982) and Pitard (1989):

$$TE=\overset{\text{Random errors: reduced but never eliminated}}{[NE+FE+GSE+QE1+QE2]}+\overset{\text{Bias: correct sampling procedures}}{[DE+EE+WE+PE+AE]}$$

The first five random errors can never be completely eliminated, but they can be minimized by careful design of the sampling

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system. Eliminating the last four sampling errors is possible, but if correct sampling practices are not diligently applied they can also be the source of major biases. The ten errors can be grouped into three categories, each of which identifies the factors affecting them most:

- The material variation (short-range variation) Errors 1, 2 and 3
- The sampling process (long-range and periodic variations) Errors 4 and 5
- The tools and techniques (includes handling) Errors 6, 7, 8, 9 and 10

$$TE = \overset{\text{Material variation}}{[NE+FE+GSE]} + \overset{\text{Process variation}}{[QE1+QE2]} + \overset{\text{Equipment and analytical variations}}{[DE+EE+WE+PE+AE]}$$

Range of error: 50%–100% 10%–20% 0.1–4%

The focus of this paper is the Fundamental Error and the three basic problems of sampling that surround this particular error. These include:

- *Problem No 1.*–What error is introduced when a sample of given weight, M_S, is taken from a pile of broken ore?
- *Problem No 2.*–What weight of sample should be taken from a pile of broken ore, so that the sampling error will not exceed a specified variance?
- *Problem No 3.*–What degree of crushing or grinding is required in order to achieve a specified value for the error variance?

Components of the Fundamental Error (FE)

The Fundamental Error (FE) variance σ^2_{FE} identified by Gy (1982) is the 'irreducible minimum' of sampling errors, is the only error that can be estimated before performing the sampling (Petersen *et al.*, 2002), and arises from the inherent variability of the material being sampled. According to François-Bongarçon (1998), FE is 'the smallest achievable residual average error', a loss of precision inherent in the sample due to physical and chemical composition as well as particle size distribution. It arises because of two characteristics of broken ore materials, namely the compositional heterogeneity and the distributional heterogeneity.

- *Compositional heterogeneity*—is a reflection of the differences in the internal composition between individual fragments of sampled ores, because of the way they are constituted and composed. The greater the difference in composition between individual fragments, the greater the compositional heterogeneity (Pitard, 1993). The terms compositional heterogeneity and constitutional heterogeneity are used interchangeably in the literature.
- *Distributional heterogeneity*—represents the difference in average composition of the lot from one place to the next in the lot; it is responsible for the irregular distribution of grade and values in groups of fragments of broken ore. The distributional heterogeneity can be influenced by large differences in density and fragment composition.

Eliminating the FE is not possible because ores are not of uniform structure or composition throughout; everything is heterogeneous, even if only at the molecular level (Bongarçon, 1995). FE arises because of the compositional and distributional heterogeneity, both factors acting to prevent the sample being representative of the whole rock pile. As indicated by Equation [1], FE is proportional to the cube of the nominal particle size and inversely proportional to the mass of the sample. Reduction of FE is therefore achieved by decreasing the diameter of the largest particles or by increasing the mass of the sample. We can, however, calculate the variance associated with the FE and hence the appropriate mass of the sample required; thus determination of the mass required for a sample is an important consideration. According to Gy (1982), if the mass of the pile of broken ore (100s of kg) is very large compared to the sample mass (a few kg), the variance of the fundamental error σ^2_{FSE} may be expressed as

$$\sigma^2_{FE} = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) fgcld_N^3 \qquad [1]$$

If the complex looking term *fgcl* is made equal to *K*, it is possible to simplify the equation and write it as:

$$\sigma^2_{FE} = \left(\frac{1}{M_S} - \frac{1}{M_L}\right) Kd_N^3$$

Where:

M_S Mass of sample measured in grams.

M_L Is the mass of material from which *Ms* is taken, measured in grams.

K Is a constant for any given ore and is the product of *c, f, g,* and *l*. *K* = *cfgl* or *K'* = *cfg*

d_N^3 Nominal size of fragments in the sample raised to the power of three

In all practical cases, M_S is much, much smaller than M_L, and Equation [1] then takes the simplified form:

$$\sigma^2_{FE} = \frac{Kd_N^3}{M_S} \qquad [2]$$

Let's return to the term *fgcl and* examine these factors.

Components of *K*

f Shape factor

f is a shape factor that relates the volume and the diameter of the particles of broken ore to one another. *f* = 0.5 (approximately π/6) that transforms the cube of the fragment size d_N^3 into the fragment volume.

$$V = \tfrac{4}{3}\pi r^3$$

$$r^3 = \frac{3V}{4\pi} \text{ and since } r^3 = \left(\frac{d}{2}\right)^3$$

$$\left(\frac{d}{2}\right)^3 = \frac{3V}{4\pi}$$

$$d^3 = \frac{6V}{\pi}$$

$$\frac{V}{d^3} = \frac{\pi}{6}$$

$$So\ f = \frac{\pi}{6} = \frac{3.1416}{6} \approx 0.5$$

The product $V_N = fd_N^3$ is the average volume of fragments of nominal size d_N (Bongarçon, 1998). The particle shape factor f is an index varying between 0.1 (needles) and 1.0 (cubes), but rarely exceeds the range 0.3–0.5. In practice, most values lie between 0.2 and 0.5, with the actual value depending on the degree to which the sample has been crushed or pulverized. For most ores a value of $f = 0.5$ is used. In very fine gold ores, $f = 0.2$ and according to Pitard (1993), it is the shape of the metal fragment responsible for the sample variability that is most important, so that beyond liberation it is the shape of the metal grains rather than the gangue that matters.

g Granulometric factor

g is defined as the average fragment volume ($\bar{V}$) divided by the nominal fragment volume (v_N) in the ratio $g = \bar{V} / v_N$ which is dependent on the reject percentage x used to define d_N. When the average fragment volume v is multiplied by g it becomes the average all-size fragment volume in the lot to be sampled. The particle size range factor g is also known as the grain size distribution factor (or the size range factor) and takes values between 0 and 1; low values of g denote a large range of particle sizes and high values denote a small range ($g = 1$ denotes all particles are of identical size).

When rock is crushed the product is a full distribution of fragment sizes, which makes a single numerical size parameter that characterizes the whole fragment size distribution d_N somewhat unrealistic. Gy examined the behaviour of the factor g as a function of the reject percentage (x) for 114 combinations of crushing equipment and materials. Six of these experimental curves are plotted in Figure 1.

The values for the reject percentage close to 5% have minimal variability around a value of g that is reasonably close to 0.25 for most ores and can be estimated from the ratio of the nominal top size d to the lower size limit d' (about 5% undersize) as follows:

➤ Large size range (no crushing)	$(d/d' > 4)$	$g = 0.25$
➤ Medium size range	$(2 \le d/d' \le 4)$	$g = 0.50$
➤ Small size range	$(d/d' < 2)$	$g = 0.75$
➤ Uniform size (pulverized)	$(d/d' = 1)$	$g = 1.00$

This approach to establishing a value for g the granulometry factor has been investigated in different ways by a number of researches, but consistently achieves values of about 0.25. According to François-Bongarçon (1998) away from the 5% reject nominal size the formula for the FE:

i. is unusable;
ii. the effect of sizing on sample precision cannot be objectively assessed, and;
iii. comparing procedures across comminution devices or types of material becomes a futile exercise.

The 5% reject size (P95, for 95% passing) has with time become the industry standard for fragment sizing. The lognormal fragment size simulation for the usual range of fragment sizes is shown in Figure 2.

c Mineralogical composition factor

c is a factor for a material consisting of two components, approximately equal to the ratio of metal density (ρ) to the dimensionless grade of the lot ('a' in 'per unit'). In its simplest form the factor has the form:

$$c = \frac{(1-a)}{a}\left[(1-a)\rho_m + a\rho_g\right]$$

For gold ores of very low grade where $a \ll 1$ it is possible to use the approximation that $c \approx \rho_m/t$, where t = gold grade ($g/t = g/1000000g$). Because a grade of 1 ppm gold is the same as $1g/t$, which is equivalent to1 gram in 10^6 grams, the mineralogical composition factor c for a gold grade of 1 g/t and a gold density of 19.3 g/cm^3 is:

$$c \approx \frac{density}{grade} = \frac{19.3}{1/1000000} = 19.3 \times 10^6 g/cm^3$$

c may seem very large, but despite the units it is not a density but, in rough terms the product of a density and a large relative variance. Generally the mineralogical composition factor is given by:

$$c = \frac{1-a}{a}\left[(1-a)\rho_m + a\rho_g\right] \text{ or}$$

$$c = \frac{(1-a)\left[(1-a)\rho_m + a\rho_g\right]}{a}$$

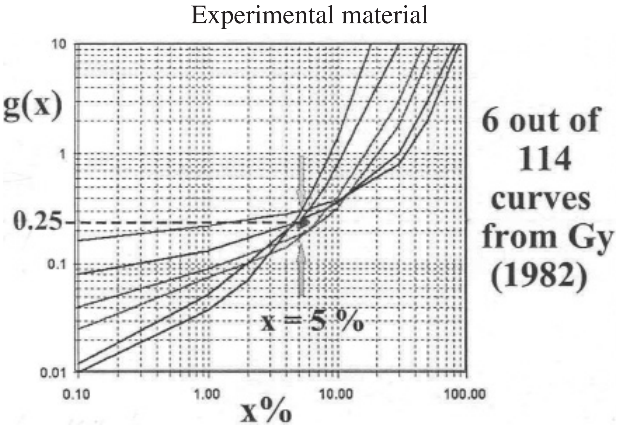

Figure 1. Gy's experimental grain size versus sieve size rejects (*x*). Source: François-Bongarçon, D. 1995. Course notes from a course entitled 'Sampling in the mining industry: Theory and Practice'

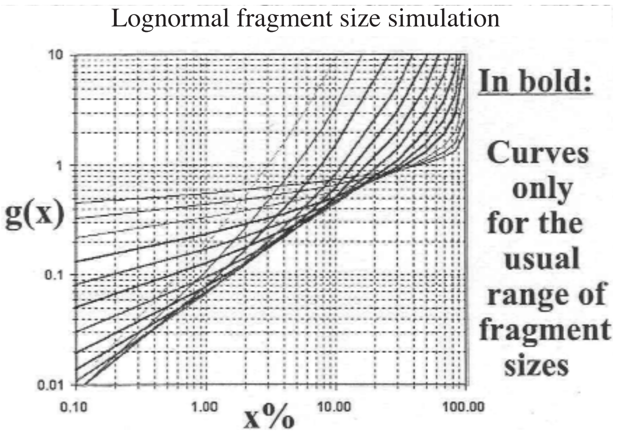

Figure 2. Comparison of fragment size and sieve size. Source: François-Bongarçon, D. 1995. Course notes from a course entitled 'Sampling in the mining industry: Theory and Practice'

where:

ρ_m = density of the metal (valuable constituent) of interest (g/cm-3).

ρ_g = density of the gangue (g/cm3).

a The decimal proportion or fractional concentration of the metal of interest

a is a factor that captures the influence of the relative proportions of the metals of interest on the mineralogical composition factor, and is calculated in the following way:

- Zinc giving an assay of 5% and occurring as sphalerite (ZnS) would have a decimal proportion of sphalerite of:

$$a = \frac{Zn+S}{Zn} \times \frac{\%ZnS}{100}$$

$$a = \frac{65+32}{65} \times \frac{5}{100} = 0.075$$

- Copper giving an assay of 0.35% and occurring as chalcopyrite would have a decimal proportion of chalcopyrite of:

$$a = \frac{Cu+Fe+2(S)}{Cu} \times \frac{\%CuFeS_2}{100}$$

$$a = \frac{63.55+55.85+2(32.06)}{63.55} \times \frac{0.35}{100} = 0.0101$$

This also means through this factor the sample variance depends on the grade of the lot being sampled, and that any use of the formula or any sampling nomogram derived from it, makes sense only when the grade level at which it is established is duly stated (François-Bongarçon, 1993 and 1995, p. 477; Pitard, 1993).

Thus at 1 ppm

$$\frac{1}{1000000} \times 100 = 0.0001\%$$

and the mineralogical factor is calculated as follows:

$$c = \frac{1-a}{a}\left[(1-a)\rho_m + a\rho_g\right]$$

$$= \frac{1-0.000001}{0.000001}\left[(1-0.000001)\right] \times 19.3 + 0.000001 \times 2.75$$

$$= \frac{0.99999}{0.000001}\left[0.999999 \times 19.3 + 0.0000275\right]$$

$$= 999999\left[19.299999 + 0.0000275\right]$$

$$c = 19.3 \times 10^6\, g/cm^3$$

At 100 ppm $c = 19.3 \times 10^4$ g/cc and at 1 000 ppm, $c = 19.3 \times 10^3$ g/cc. More often than not the approximation that $c \approx \rho_m/t$ can be used for low concentration ores. In fact the value of *c* for gold is generally lower because most of the gold occurs as a gold-silver amalgam that according to Pitard (2006, personal communication) has a density of around 16 g/cc in which case *c* = 16 000 000.

In the general case the density of the crushed ore varies with the degree of comminution and is a function of the proportions of metal and gangue. So the product *cl* is not a variance multiplied by a density, but is rather a density-weighted variance in which each fragment's contribution to the total variance is weighted by its density (François-Bongarçon, 1995; Pitard, 1993).

ℓ Liberation factor

ℓ is a dimensionless number between 0 (no liberation) and 1 (complete liberation), which varies with the size of the fragments and also depends on the nominal size of the fully liberated metal grains. It also depends on the geostatistical characteristics of the mineralization at microscopic scale, i.e. spatial correlations within the fragments. The liberation size *d* is the nominal size at which the fragments of the lot must be crushed so that the mineral grains become fully liberated from the gangue and at the liberation size (as well as below) the liberation factor is equal to one. This is an ideal concept and for practical purposes, it is the size at which approximately 85 per cent of the large fragments have been liberated. The use of the value 0.5 for the exponent in this equation has been the subject of some considerable discussion. According to François-Bongarçon (1998, 1999), and François-Bongarçon and Gy (2002), Gy's empirical liberation factor for unliberated particles $\ell = \sqrt{\frac{d_\ell}{d_N}}$ also written as $\ell = \left(\frac{d_\ell}{d_N}\right)^{0.5}$ does not give a good result and he has suggested a more general form for the liberation factor provides a better result as given here:

$$\ell = \left(\frac{d_\ell}{d_N}\right)^b \qquad [3]$$

By contrast, Pitard (1993) fully supports Gy's use of the 0.5 as the exponent in Equation [3] saying that it depends on how it is used and what the state of the broken ore is.

b

Is a value related to the slope of the calibration line above the liberation size d_l. The value for *b* can vary between 0 and 3 depending on the nature of the ore and requires calibration to a particular ore type (de Castilho *et al.* 2005). Exponent *b* in Equation [3] takes values close to 1.5 in most gold ores as well as in cases where it has not been possible to calibrate the exponent and so we can write:

$$\ell = \left(\frac{d_\ell}{d_N}\right)^{1.5}$$

Using a liberation factor *b* = 1.5 it is possible to produce sampling nomograms that are realistic, correct and useful.

d_N

Nominal size of fragments in the sample, is equivalent to the maximum particle size in the lot to be sampled. In practice, d_N is taken to be the screen size that retains 5% of the lot being sampled. For example, if a sample is sieved using a 2.5 cm aperture screen and 5% of the sample is retained on top of the sieve then d_N = 2.50 cm. Note: In RSA sieves are referred to only by their aperture and although the old mesh notation is no longer used, it nevertheless appears on the screen name plate.

d_ℓ liberation size and liberation factor

There is a change in the form of the relative variance of the FE when the ores become fully liberated (see Figure 3.)

Beyond liberation size further comminution does not change the variability of the individual rock fragments (François-Bongarçon 1998). From the log-log graph of $l(d_l)$ x

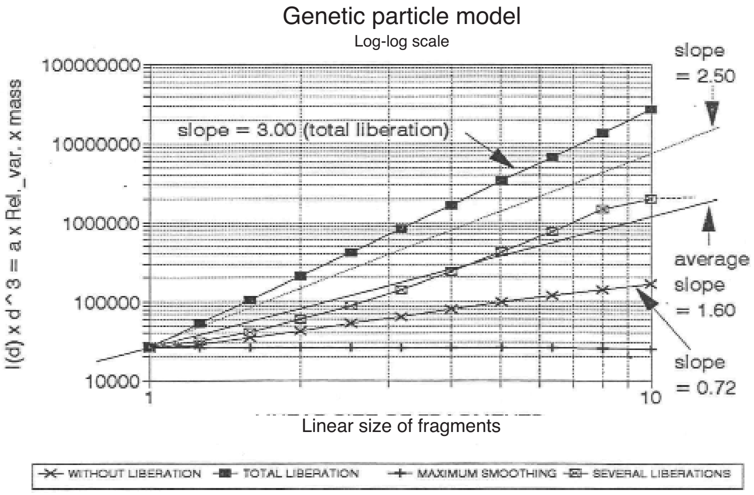

Figure 3. Comparison of linear fragment size with relative variance x mass. Source: François-Bongarçon, D. 1995. Course notes from a course entitled 'Sampling in the mining industry: Theory and Practice'

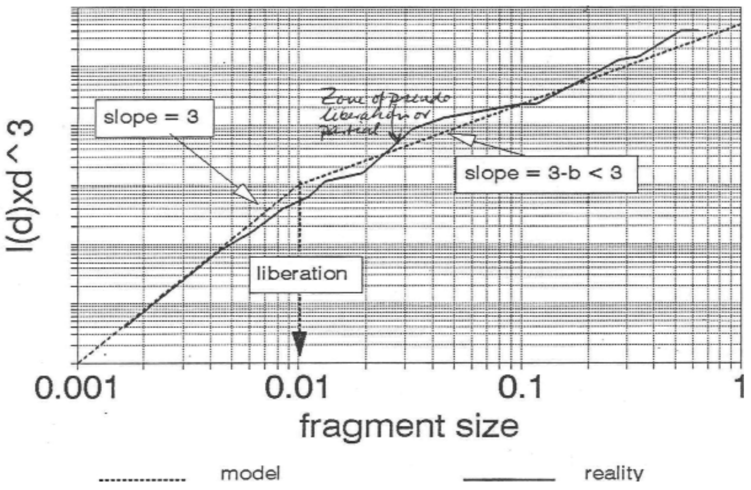

Figure 4. Fitting a general case to fragment sizes above and below the liberation size (0.01); Above the liberation size slope *a* = 3–*b*; below the liberation size slope *a* = 3. Source: François-Bongarçon, D. 1995. Course notes from a course entitled 'Sampling in the mining industry: Theory and Practice'

d^3 versus fragment size (shown in Figure 3) it is known that the slope of the line below the liberation size (d_ℓ) is equal to 3. Beyond the liberation size the slope of the line is less than 3 and is given by $\alpha = 3–b$ and the effective range for b is actually 1 to 2. According to François-Bongarçon (2004):

$$\ell = \left(\frac{d_\ell}{d_N}\right)^{b} = \left(\frac{d_\ell}{d_N}\right)^{3-a}$$

where b is a factor related to the slope of the line and varies between 0 and 3. The underlying insight to this factor is shown in Figure 4.

Applications of Gy's equation to the problems of sampling

Having derived all the components of Gy's equation it is now possible to answer the questions that were posed earlier. For the sake of this exercise assume the following sampling conditions for a gold bearing ore crushed to about 0.93 cm, with c = 16 000 000 for a gold-silver amalgam, f = 0.5, g = 0.25 and ℓ = 0.000035.

- *Problem No. 1*—What error is introduced when a sample of given weight, M_S, is taken from a pile of broken ore? This problem is simply answered by substituting the known factors into the original Gy's equation. Assume that d_N = 1.25 cm.

$$\sigma_R^2 = \frac{c \times f \times g \times \ell \times d_N^3}{M_S}$$

$$= \frac{16000000 \times 0.5 \times 0.25 \times 0.000035 \times 1.25^3}{10000}$$

$$\sigma_R^2 = 0.0875$$

$$\sigma_R = 0.2958$$

$$\text{Precision} = 29.58\%$$

A precision of 29.58% must be compatible with the data quality objectives of the sampling protocol for this value to be expected.

- *Problem No. 2*—What weight of sample should be taken from a pile of broken ore, so that the sampling error will not exceed a specified precision σ_{FE}, let us say 15%? This requires a simple rearrangement of Gy's equation in the form:

$$M_S = \frac{c \times f \times g \times \ell \times d_N^3}{\sigma_R^2}$$

and the substitution of the appropriate factors to give:

$$M_S = \frac{c \times f \times g \times \ell \times d_N^3}{\sigma_R^2}$$

$$= \frac{16000000 \times 0.5 \times 0.25 \times 0.000035 \times 0.93^3}{0.0225}$$

$$= 16015g \text{ or } 16.02\text{kg}$$

A sample mass of approximately 16 kg is required to achieve a precision of 15% for this ore type.

- *Problem No. 3*—What degree of crushing or grinding is required in order to achieve a specified value for the error variance σ_R^2? Again this requires rearrangement of Gy's equation as follows:

$$d_N^3 = \frac{M_S \times \sigma_R^2}{c \times f \times g \times \ell}$$

Assume here that the precision is 15%, equivalent to an error variance is 0.0225, and that the mass of material to be collected is 15 kg.

$$d_N^3 = \frac{15000 \times 0.06575}{16000000 \times 0.5 \times 0.25 \times 0.000035}$$

$$d_N = \left(\frac{337.5}{448}\right)^{1/3}$$

$$d_N = 0.91cm$$

The fragment size should be 95% passing 0.9 cm to achieve a precision of 15% if a sample of 15 kg is collected.

Conclusions

Although the Fundamental Error is only one of ten sampling errors that the practitioners needs to take cognisance of, the approach taken by Pierre Gy provides an excellent inferential relationship between the mass of material to be sampled, the size of the fragments being sampled and the relative variance of the sampling error. It also provides solutions for three of the main problems associated with the Theory of Sampling.

References

Petersen, L., Dahl, C.K., and Esbensen, K.H. ACABS Research Group. Correct Sampling—A Fundamental Prerequisite for Proper Chemical Analysis—I. ACABS Posters. www.acabs.dk. 2002.

Brunton, D.W. The Theory and Practice of Ore Sampling. *Transactions AIME*, vol. 836, no. 25. 1895.

De Castilho, M.V., Mazzoni, P.K.M., and François-Bongarçon, D. Calibration of parameters for estimating sampling variance. R. Holmes (ed.), *Second World Conference on Sampling and Blending*, 2005. Australian Institute of Mining and Metallurgy, Publication Series no. 4/2005. May 2005. pp. 3–8.

François-Bongarçon, D. The Practice of the Sampling of Broken Ores. *CIM Bulletin*, May 1993. vol. 86, no. 970, 1993. pp. 75–81.

François-Bongarçon, D. Sampling in the Mining Industry: Theory and Practice, Volume 1: Course notes and Transparencies. A Short Course presented by D. François-Bongarçon in the School of Mining Engineering, University of the Witwatersrand. 1995.

François-Bongarçon, D. The most common error in applying 'Gy's Formula' in the theory of mineral sampling. Available from D. François-Bongarçon, President, AGORATEK International, Palo Alto, CA, USA at www.agoratekinternational.com or Dfbgn@attglobal.net. 1998.

François-Bongarçon, D. The Modeling of the Liberation Factor and its Calibration. Available from D. François-Bongarçon, President, AGORATEK International, Palo Alto, CA, USA at www.agoratekinternational.com or Dfbgn@attglobal.net. 1999.

François-Bongarçon, D. and Gy, P. The most common error in applying 'Gy's Formula' in the theory of mineral sampling, and the history of the liberation factor. *South African Institute of Mining and Metallurgy Journal*, vol. 102, no. 8. 2002. pp. 475–479.

Gy, P.M. The sampling of broken ores—A review of principles and practice. The Institution of Mining and Metallurgy, London, *Geological, Mining and Metallurgical Sampling*: 1973. pp. 194–205, and discussion pp. 261–263.

Gy, P.M. *Sampling of Particulate Materials, Theory and Practice. Developments in Geomathematics 4*. Elsevier Scientific Publishing Company. 1979. 430 pp.

Gy, P.M. *Sampling of Particulate Materials, Theory and Practice*, Second Revised Edition. Elsevier, Amsterdam. 1982.

Gy, P.M. *Sampling of Heterogeneous and Dynamic Material Systems. Theories of heterogeneity, Sampling and Homogenising*. Elsevier, Amsterdam 1992.

Gy, P.M. Introduction to the Theory of Sampling. Part 1: Heterogeneity of a Population of Uncorrelated Units. *Transactions AC*, 14, 1995. pp. 67–76.

Gy, P.M. *Sampling for Analytical Purposes: The Paris School of Physics and Chemistry*, Translated by A.G. Royle. John Wiley and Sons, Inc. New York. 1998. 153 pp.

Pitard, F.F. *Pierre Gy's sampling theory and sampling practice. Heterogeneity, sampling correctness, and statistical process control*. CRC Press, Inc. 1993. 488 pp.

Pitard, F.F. Sampling Correctness—A Comprehensive Guide. *Second World Conference on Sampling and Blending*. Novotel Twin Waters Resort, Sunshine Coast, Queensland, Australia 10-12 May 2005. pp. 55–68.

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Appendix 1

Below the liberation size dℓ

Gy's formula can be written:

$$\sigma_R^2 = \frac{K' \times d_N^3}{M_S}$$

where $K' = c * f * g$ because below the liberation size $\ell = 1$ for all sizes.

Above the liberation size d_ℓ

For the sake of this exercise assume the following sampling conditions for 10 kg of gold-bearing ore crushed to about 0.93 cm. With *c* equal to 19 300 000, *f* equal to 0.5, *g* equal to 0.25 and ℓ equal to 0.000035, the precision of the FE is:

$$\sigma_R^2 = \frac{c \times f \times g \times \ell \times d_N^3}{M_S}$$

$$\sigma_R^2 = \frac{19300000 \times 0.5 \times 0.25 \times 0.000035 \times 1.25^3}{1000}$$

$$\sigma_R^2 = \frac{1055.47}{10000} = 0.105555$$

$$\sigma_R = 0.3249$$

$$\text{Presicion} = 32.49\%$$

In order to keep the precision below 15% either the mass of the sampled material must be increased or the nominal fragment size must be reduced to say 0.93 cm.

Gy's formula takes the form $\sigma_R^2 = \frac{c \times f \times g \times \ell \times d_N^3}{M_S}$ where α = 3–b. Since we know $\ell = \left(\frac{d_l}{d_N}\right)^b$ we can substitute for l in Gy's equation and further write:

$$\ell = \left(\frac{d_\ell}{d_N}\right)^b$$

$$\sigma_R^2 = \frac{cfgd_N^3}{M_S} \times \left(\frac{d_\ell}{d_N}\right)^b$$

$$= \frac{cfgd_N^3}{M_S} \times \frac{d_\ell^b}{d_N^b}$$

$$= \frac{cfgd_\ell^b}{M_S} \times \frac{d_N^3}{d_N^b}$$

$$= \frac{cfgd_\ell^b \times d_N^3}{M_S}$$

and because $\alpha = 3 - \text{b}$ we can write

$$\sigma_R^2 = \frac{cfgd_{\ell l}^{3-\alpha} d_N^\alpha}{M_S}$$

d_ℓ is liberation size for mineral particles, i.e. the maximum particle diameter which ensures complete liberation of the mineral. d_ℓ is measured in cm. Provided you have the liberation factor you can rearrange the equation $\ell = \left(\frac{d_\ell}{d_N}\right)^\text{b}$ in its most simple form to give us $d\ell$ as follows:

$$\ell = \left[\frac{d_\ell}{d_N}\right]^b$$

$$d_\ell = d_N \times \ell^{1/b}$$

$$d_\ell = 1.25 \times \left(3.5 \times 10^{-5}\right)^{1/1.5}$$

$$= 0.001337\text{cm or } 133.7 \text{ microns}$$

In a more complex format we can derive the liberation size as follows:

An alternative arrangement of the equation for the liberation size is as follows:

$$\sigma_R^2 = \frac{cfg \times d_\ell^{(3-\alpha)} \times d_N^\alpha}{M_S}$$

$$0.01 = \frac{1}{10000} \times 19300000 \times 0.5 \times 0.25 \times d_l^{1.5} \times 1^{1.5}$$

$$0.01 = 241.25 \times d_\ell^{1.5}$$

$$d_\ell = 0.000041451^{\frac{1}{1.5}}$$

$$d_\ell = 0.0011977\text{cm}$$

$$d_\ell = 119.8 \text{ say } 120\ \mu\text{m}$$

The general case for Gy's formula can be written as

$$\sigma_{FE}^2 = \frac{c \times f \times g \times d_l^{3-\alpha} \times d_N^\alpha}{M_S} \qquad [4]$$

where:

α = a parameter for specific deposits which can be 'calibrated' to a particular ore type. Current research has indicated that α = 1.5 for most low grade gold ores (see[1]). Equation [4] can be rearranged so that

$$M_S = \frac{c \times f \times g \times \ell \times d_N^3}{\sigma_{FE}^2} \qquad [5]$$

We now have all the tools necessary to answer the three questions stated at the beginning of the appendix.

ERRATUM

Please note on pages 137 and 138 of the February *Journal* 2007, in the Comments:

Mining method selection by multiple criteria decision making tool

and

EQS: a computer software using fuzzy logic for equipment selection in mining engineering

By: M. Yavuz and S. Pillay

The author's name should read S. Alpay and not S. Pillay