Learning about Commodity Cycles and Saving-Investment Dynamics in a Commodity-Exporting Economy*

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Despite high levels of commodity prices, the current accounts of several commodity exporters have deteriorated or even reverted recently. This phenomenon is examined using a quantitative small open-economy model with a commodity sector and with imperfect information and learning about the persistence of commodity price shocks. The model predicts that during a persistent commodity price increase, agents believe at first that this increase is temporary but then revise their expectations upward as they are surprised by higher effective prices. Investment therefore expands gradually, driven by the commodity sector, while domestic savings fall, which explains the observed current account dynamics.

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1. Introduction

Commodity prices have surged over the past decade, interrupted only briefly by the global financial crisis. Most commodity exporters have thus enjoyed record or near-record terms of trade during a prolonged period. Despite this fact, many commodity-exporting countries have accumulated significant current account deficits that have become an important policy concern for several reasons, such as the risk of a painful adjustment in the face of a sudden stop in capital flows. Some countries have even experienced current account reversals from positive balances in the mid-2000s into deficits of several percentage points of GDP in recent years. In Chile, a major copper producer, the reversal has been especially large.

What explains this seemingly surprising pattern of current account dynamics in commodity-exporting economies? The observed dynamics appear at odds with the response of the current account to a positive terms-of-trade shock that may be expected according to standard macroeconomic theory under full information, which predicts that temporary income gains would be mostly saved by rational agents, while permanent income gains would be immediately and entirely spent. That theory is well known as the intertemporal approach to the current account (see Obstfeld 1982; Svensson and Razin 1983; Mendoza 1995; Obstfeld and Rogoff 1995). However, Obstfeld (1982) and Svensson and Razin (1983) also pointed out that the effect of a terms-of-trade shock on the trade balance should depend on the *perceived* persistence of the shock. The latter points towards a potentially important role for imperfect information on the persistence of terms-of-trade changes and learning behavior by economic agents.

Hence, this paper examines the role of learning under imperfect information on the persistence of commodity price shocks for saving-investment dynamics in a dynamic stochastic general equilibrium (DSGE) model for a commodity-exporting small open economy. Unlike existing models of this type, our framework postulates that agents cannot perfectly distinguish between persistent and transitory movements in the price of the exported commodity. Instead, agents learn over time about the true persistence of the shock. Technically, agents use optimal filtering to update their inference on the persistence of the shock based on past forecasting errors.

We provide evidence that supports the practical relevance of this hypothesis by analyzing actual revisions of forecasts made by expert forecasters, based on a simple statistical unobserved-components model for commodity prices which is then embedded into the DSGE model.

The DSGE model predicts that if the persistence of a commodity price shock turns out to be high, then the materialization of higher expected returns of capital—especially in the commodity sector—triggers an investment boom. Under learning, the lagged response of commodity investment, which is subject to adjustment costs and time-to-build frictions, is key to the eventual appearance of a current account deficit after an initial surplus caused by an increase in savings in the early phase of the commodity price surge (while agents think it is short-lived). We conduct a Bayesian estimation of the model using Chilean data to assess the quantitative significance of this mechanism. We find that a significant fraction of recent investment and current account dynamics are explained by commodity price movements through that mechanism. Thereby, agents' perceptions on the persistence of commodity price cycles and their interaction with investment in the commodity sector are validated empirically as important aspects to explain recent current account dynamics in commodity-exporting economies such as Chile.

A few previous studies have examined the key drivers of the current account in commodity-exporting countries using quantitative DSGE models. For example, Medina, Munro, and Soto (2008) estimate a DSGE model modified for Chile and New Zealand and show that the main factors behind current account fluctuations in both countries are aggregate investment-specific shocks and changes in foreign financial conditions and foreign demand.

One of our contributions is to allow for endogenous production decisions in the commodity sector. Production is carried out with capital, subject to time to build and adjustment costs in investment projects following Kydland and Prescott (1982) and Uribe and Yue (2006), which play a key role under imperfect information. The recent boom of mining investment in most commodity-exporting countries—through foreign direct investment by international mining companies or state-owned enterprises—emphasizes the relevance of this channel. In addition, the introduction of time to build accounts for the fact that the majority of investment projects in the mining sector are large and thus take several quarters or years to mature.

A second main contribution of this paper is to implement imperfect information and learning through the Kalman filter on the persistence of commodity price shocks, drawing upon studies on the role of imperfect information and optimal filtering in macroeconomic dynamics following Erceg and Levin (2003).[1] This application is motivated by evidence of gradual revisions of commodity price forecasts by chief international forecasting institutions and, in the case of Chile, by the panel of experts that determines the long-run reference price of copper, a variable that enters Chile's structural balance fiscal rule. As we document, in the initial phase of the latest commodity cycle, the forecasters predicted that commodity prices would revert relatively quickly towards their long-run means. But as the actual price increases turned out more persistent, the forecasts were gradually revised upwards. After the cycle turned more recently, it again took several years for price forecasts to be adjusted downwards. We show that those gradual forecast revisions are consistent with the mechanism of optimal filtering that we apply in our model.

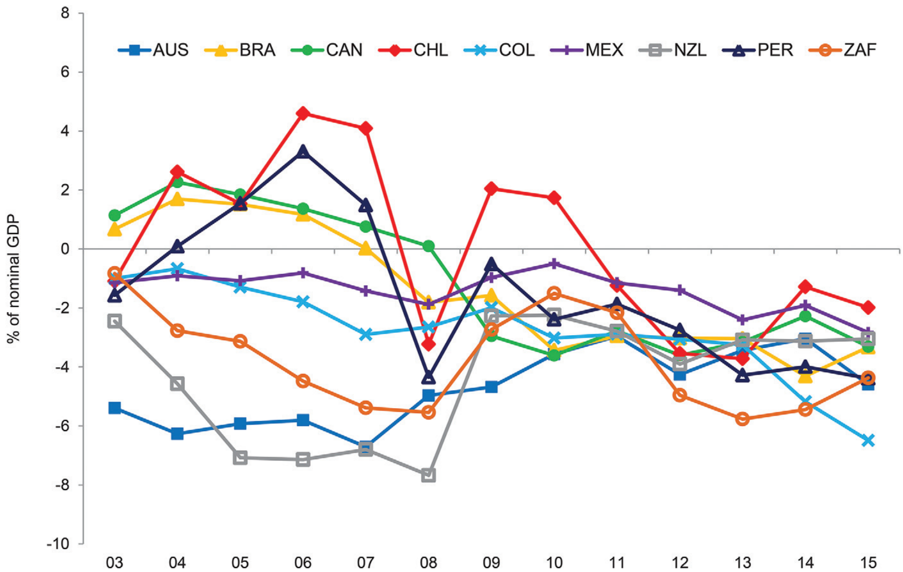
The remainder of the paper is structured as follows. Section 2 discusses some stylized facts regarding the evolution of commodity price forecasts as well as investment and current account dynamics in Chile and other major commodity exporters that we seek to match. Section 3 discusses the implementation of imperfect information and learning using a simple unobserved-components model, section 4 describes the DSGE model embedding that simple model and the estimation, section 5 presents the results from the DSGE model, and section 6 concludes.

2. Stylized Facts

This section discusses a number of key stylized facts to be matched. These facts concern recent current account dynamics in Chile and other major commodity exporters, as well as the evolution of expert forecasts for different commodity prices (at the time of writing of this paper in 2016).

[1]See Céspedes and Soto (2007) for an application to inflation dynamics.

Figure 1. Current Account Balances of Major Commodity-Exporting Countries, 2003–15



Data Source: International Monetary Fund (IMF), World Economic Outlook Database, April 2016. The numbers for Brazil, Chile, Colombia, and South Africa for 2015 are IMF estimates.
Note: This figure shows the annual current account balances as percentages of nominal GDP from 2003 to 2015 of Australia (AUS), Brazil (BRA), Canada (CAN), Chile (CHL), Colombia (COL), Mexico (MEX), New Zealand (NZL), Peru (PER), and South Africa (ZAF).

2.1 Saving-Investment Dynamics in Chile and Other Commodity Exporters

Figure 1 displays the recent evolution of the current account balances of major commodity exporters.[2] While all countries have exhibited current account deficits in recent years, some countries—such as Brazil, Canada, Chile, and Peru—have actually experienced reversals from positive balances of several percentage points of GDP in the mid-2000s to deficits of more than 2 percent of GDP more

[2] All figures can be seen in color in the online version, available at http://www.ijcb.org.

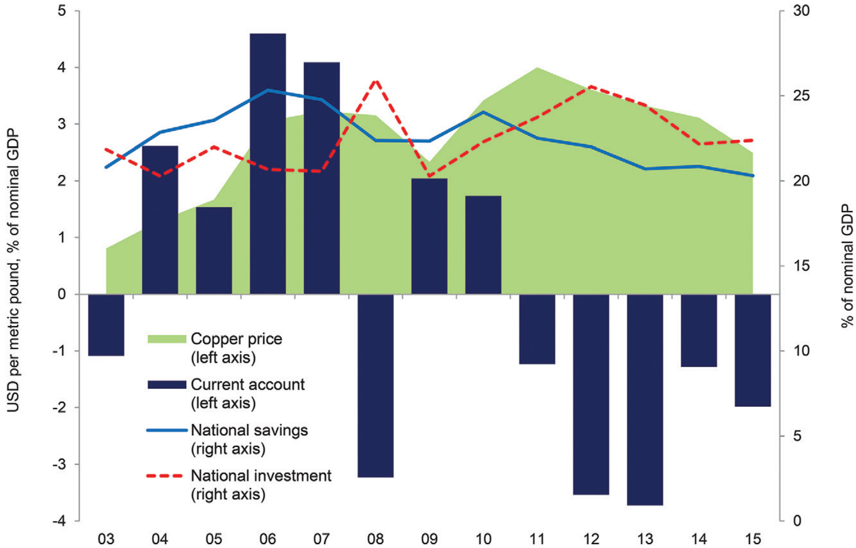
recently. The reversal has been especially large in the case of Chile, up to around 8 percentage points. These dynamics point towards a potential common explanation. Indeed, the evolution of the current accounts of most countries shown has been closely linked to the behavior of mining investment. In Fornero, Kirchner, and Yany (2015), we provide cross-country evidence on this aspect. In what follows, we focus on the experience of Chile.[3]

Chile has exhibited a large current account deficit over the past few years, despite relatively high levels of commodity prices and, in particular, the price of copper. Figure 2 plots the evolution of annual national savings, fixed capital formation, and the current account balance as a percentage of nominal GDP against the average annual spot price of refined copper at the London Metal Exchange over the period 2003–15. The Chilean current account has moved from a deficit of around 1 percent of GDP in 2003 to a surplus of more than 4 percent in 2006 and 2007, at a time when the copper price quadrupled from 80 U.S. dollar cents per metric pound in 2003 to 320 cents in 2007. But after the global financial crisis, the current account gradually deteriorated despite the strong and sustained recovery of the copper price, reaching a deficit of more than 3 percent of GDP after 2011. This pattern can also be observed in figure 1 for other countries.

The peculiar dynamics of the Chilean current account can be explained by the evolution of the gap between national savings and investment. The current account surplus during the mid-2000s was generated by a rise in national savings, while national investment remained steady at a share of GDP of roughly 21 percent. However, from 2008 onwards investment started to increase, reaching approximately 25 percent of GDP in 2012. On the other hand, national savings declined gradually over that period from almost 25 percent of GDP in 2006 to less than 22 percent of GDP in 2012, such that the current account moved into deficit in a stepwise fashion. Hence, while the gains from commodity exports seem to have been mostly saved by domestic agents in the mid-2000s period, this was not the case in the post-crisis period.

[3]Similar evidence for Peru is discussed in Central Reserve Bank of Peru (2016). See also International Monetary Fund (2015) and Magud and Sosa (2015) for other related cross-country evidence.

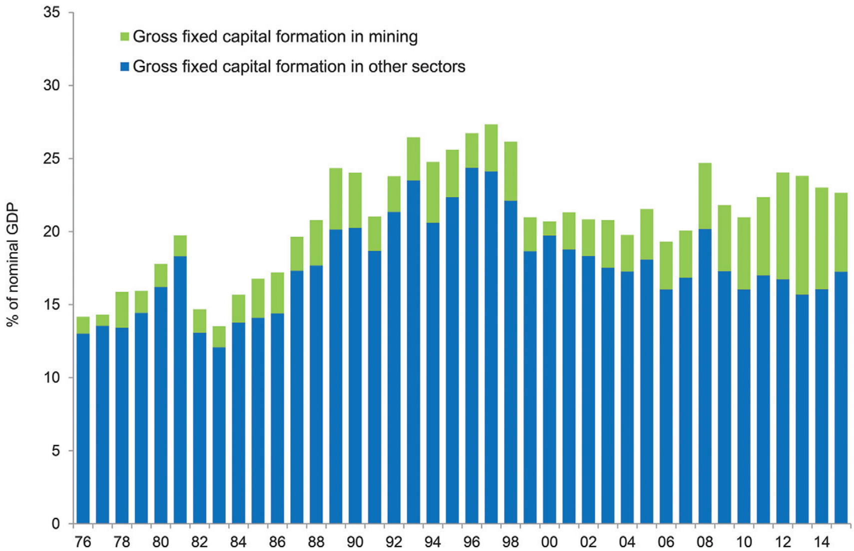
Figure 2. Savings, Investment, and the Current Account in Chile versus Copper Price, 2003–15



Data Sources: Central Bank of Chile and International Monetary Fund, World Economic Outlook Database, April 2016.
Notes: This figure shows annual national savings (solid line, right axis), fixed capital formation (dashed line, right axis), and the current account balance (bars, left axis) in Chile from 2003 through 2015. All variables are expressed as percentages of nominal GDP. The shaded area corresponds to the average annual spot price of refined copper at the London Metal Exchange in U.S. dollars per metric pound (left axis).

Most of the investment boom in Chile after 2008 has been due to a relatively large increase in mining investment. Figure 3 shows the evolution of gross fixed capital formation in mining and other sectors as a percentage of GDP over the period 1976–2015. Investment in mining was on average less than 2 percent of GDP from 1976 until the mid-1980s and around 3 percent from the second half of the 1980s until the mid-2000s. However, starting in 2008, mining investment has increased significantly to more than 8 percent of GDP in 2013. Only very recently has investment in mining started to decrease again, reaching approximately 7 percent and 5 percent of GDP in 2014 and 2015, respectively. Non-mining investment has remained comparatively stable compared to its pre-2008 levels.

Figure 3. Gross Fixed Capital Formation in Mining and Other Sectors in Chile, 1976–2015



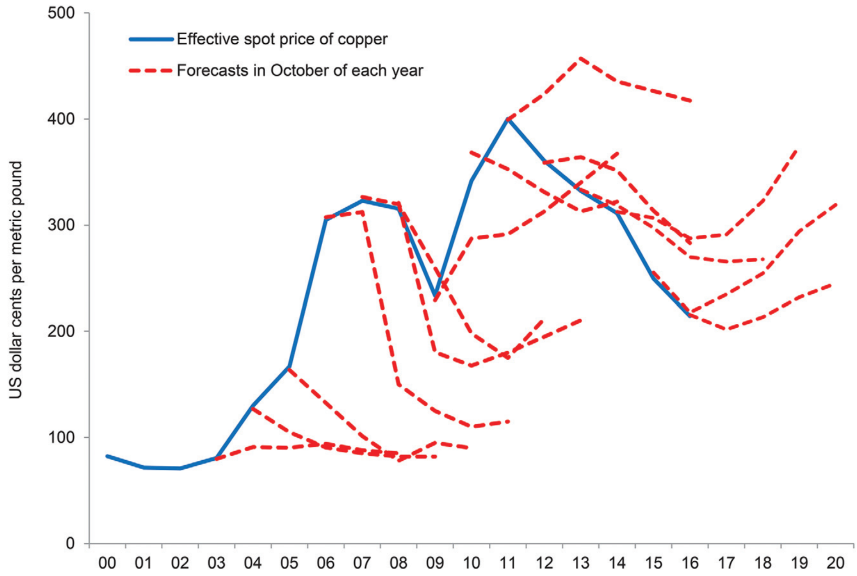
Data Sources: Chilean State Copper Commission (Cochilco) and Central Bank of Chile. The sectoral division of investment for 2015 is a preliminary estimate from the Central Bank of Chile's Monetary Policy Report (September 2016).
Note: This figure shows total gross fixed capital formation in the mining sector and other sectors in Chile from 1976 through 2015, expressed as percentages of nominal GDP.

An equally important role of mining investment has been detected in other major commodity exporters, such as Australia (see Tulip 2014) and Peru (see Central Reserve Bank of Peru 2016).

2.2 Commodity Price Forecasts and Forecast Revisions

We now examine the evolution of copper price forecasts by professional forecasters of the CRU Group (reports in October of each year), as shown in figure 4. The rise of the spot price in the mid-2000s was not validated by higher forecasted prices on a medium- to long-term horizon. Instead, it was considered as a transitory price increase by the professional forecasters who predicted that the spot price would return to values of around 100 cents. Due to the crisis,

Figure 4. Spot Price of Copper from 2000 through 2006 versus Forecasts by CRU Group



Data Sources: Central Bank of Chile and CRU Group.
Notes: This figure shows the average annual spot price of refined copper at the London Metal Exchange in U.S. dollar cents per metric pound from 2000 through 2016 (solid line) and forecasts of the spot price by professional forecasters of the CRU Group made in October of each year starting in 2003 (dashed lines). The spot price for 2016 is the average price from January to October 2016.

the price fell and almost reversed the rise from 2003 to 2007, reaching a minimum of approximately 140 cents (where the annual average understates somewhat the dynamic evolution of the spot price). That decline was relatively short-lived, and after the crisis the copper price quickly recovered and exceeded its pre-crisis levels. However, the higher post-crisis prices were also accompanied by higher forecasted prices, as part of a process of gradual forecast revisions that had already started around 2007. In the following years, the forecasted prices reached values much closer to the effective spot price. Hence, the professional forecasters seem to have incorporated a more persistent price increase in their forecasts over time. More recently, as the commodity cycle has turned and prices have fallen, it has taken the forecasters again several years to adjust their expectations on future prices downwards.[4]

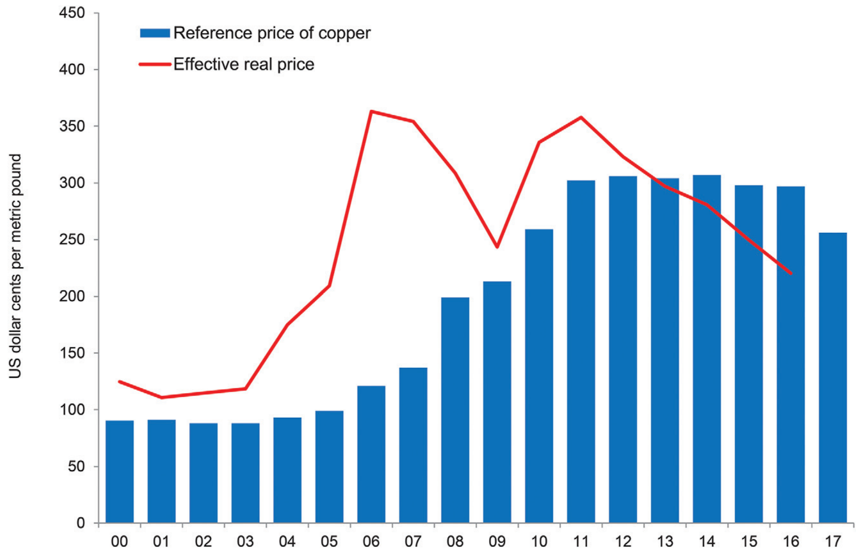
The previous analysis can be complemented by examining the reference price of copper used by the Chilean Ministry of Finance. The reference price is computed from long-run forecasts of future spot prices over a ten-year period by a panel of independent experts.[5] As such, it is one of the main variables of Chile's fiscal rule. Figure 5 plots the evolution of the reference price, which by construction is expressed in real terms, against the effective real price of copper, for the fiscal years starting in 2000 until 2017. As with the professional forecasts discussed above, the reference price increased gradually over time. Remarkably, the forecasts after 2011 were not too far from the spot prices in the previous years, so that the experts had incorporated sustained high levels of copper prices in their forecasts by that time. Moreover, the reference price has only started to decrease very recently from 2015 onwards, despite the fact that effective prices had already been decreasing for some time. Only the most recent forecast for 2017 (made in 2016) was more heavily revised downwards.

Due to the characteristics of the Chilean fiscal rule, which adjusts the government budget by the gap between the reference price and the actual price of copper, this stepwise increase in the reference price gradually allowed for more public spending out of copper revenues, both from income of the state-owned copper company (Codelco) and from taxes on private copper producers. Hence, while the government mainly saved the additional copper revenues during the initial stages of the boom in the mid-2000s, which added to the overall current account surplus during that period, it decreased its savings over time, contributing to the gradual deterioration of the current account balance. Changes in perceptions on the persistence of the commodity cycle again played a key role in this process.

[4]Comelli and Ruiz (2016) report very similar evidence based on International Monetary Fund forecasts for copper from 1993 to 2015.

[5]The panel of experts comprises sixteen members. Each member is asked to submit forecasts of the annual (real) spot price for the following ten years (in U.S. dollar cents of the following fiscal year, using as deflator the forecast of the U.S. CPI by the IMF). The forecasts of each expert are averaged over time and the two most extreme values—highest and lowest—are discarded. The reference price is the average of the remaining ten-year averages.

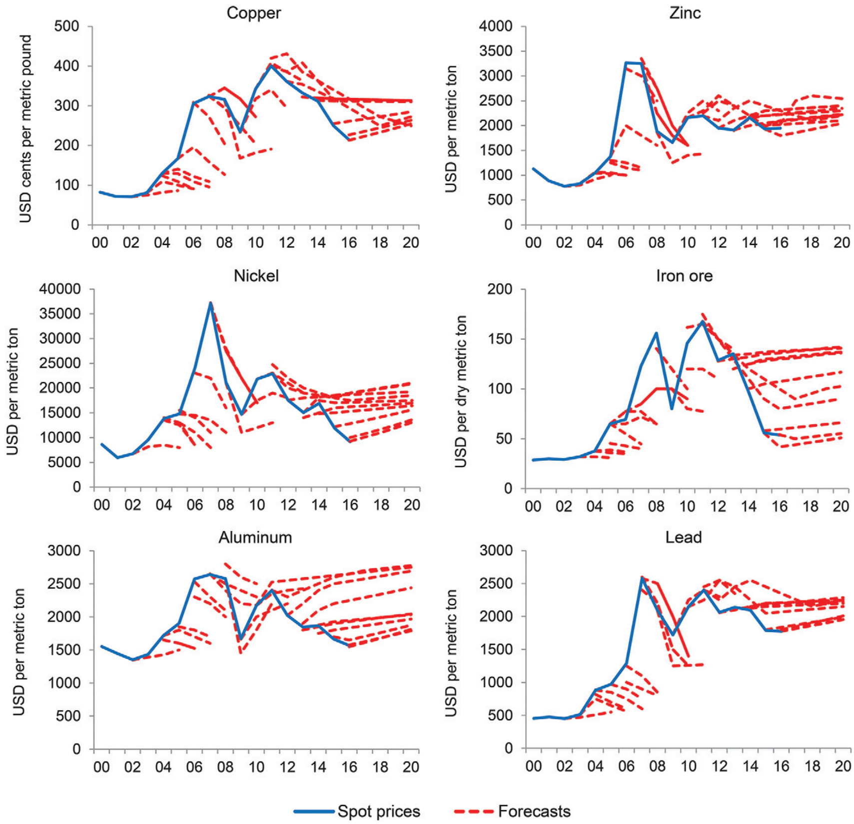
Figure 5. Reference Price versus Effective Real Price of Copper, 2000–17



Data Sources: Chilean Ministry of Finance and Central Bank of Chile.
Notes: This figure shows the average annual spot price of refined copper at the London Metal Exchange in U.S. dollar cents per metric pound deflated by an index of external prices published by the Central Bank of Chile ("IPE") which was converted into an index with base year 2015 (solid line) and the reference price of copper used by the Chilean government for 2000 through 2017 (bars). The spot price for 2016 is the average price from January to September 2016.

To close this section, figure 6 extends the previous evidence for copper by documenting forecast revisions for a wider set of commodities (metals) based on World Bank forecasts starting in the early 2000s. These commodities include zinc, nickel, iron ore, aluminum, and lead, as well as copper. For all of them, the forecasts incorporated a relatively rapid return towards the respective historical averages in the early phase of the 2000s commodity boom but were then revised gradually, incorporating more persistent price increases. As commodity prices have fallen more recently, price forecasts have again gradually been adjusted downwards. This evidence points towards a common explanation such as the one emphasized in this paper.

Figure 6. Spot Prices of Various Commodities from 2000 through 2016 versus World Bank Forecasts



Data Sources: International Monetary Fund and World Bank.
Notes: This figure shows the average annual spot prices of different commodities at the London Metal Exchange in U.S. dollars and in the respective units from 2000 through 2016 (solid lines) and forecasts of the spot prices by the World Bank from various reports (different months) starting in 2003 (dashed lines). The spot prices for 2016 are the average prices from January to October 2016.

2.3 Summary of Stylized Facts

We have documented the following three stylized facts related to current account dynamics in commodity-exporting countries and the evolution of commodity price forecasts over the latest commodity cycle: (i) sustained high levels of commodity prices gradually led to higher forecasted prices; (ii) high spot prices and higher forecasted prices were associated with current account reversals from surplus to deficit in some countries—including Brazil, Canada, Chile, and Peru—and more negative balances in others, such as Colombia and Mexico; (iii) the observed investment boom has been driven by mining in most countries analyzed.

These observations point towards the importance of changing perceptions by market agents on the persistence of commodity prices, a hypothesis that we investigate formally in the following sections. These changing perceptions will be critical to understand, according to the analysis conducted, the recent evolution of mining investment, and the current account in Chile and other commodity exporters. More precisely, for the market it took between four and five years to adjust their expectations regarding the persistence of the 2000s commodity price increase. While expectations were revised upwards, in parallel mining investment increased significantly, which contributed to the observed deterioration of current account balances. Recently, despite the observed reversal of commodity prices, mining investment remained stable as long as price forecasts remained relatively high.

3. Learning about Commodity Cycles

This section describes the formal implementation of imperfect information and learning about the persistence of commodity price cycles, based on a simple statistical model for the commodity price with persistent and transitory shocks. The model is applied for copper and estimated using data on spot prices as well as expert forecasts to identify the historical shocks.

3.1 A Simple Unobserved-Components Model

Let $p^*_{S,t} = P^*_{S,t}/P^*_t$ be the real international commodity price and let $\breve{p}^*_{S,t}$ denote the log-deviation of $p^*_{S,t}$ from its unconditional mean $\bar{p}^*_S$, i.e., $\breve{p}^*_{S,t} = \log(p^*_{S,t}/\bar{p}^*_S)$. We introduce imperfect information on the persistence of commodity prices, assuming that $\breve{p}^*_{S,t}$ has two different components: a transitory component, a_t, and a persistent component, b_t. The dynamics of the persistent component may be interpreted to capture changes in economic "fundamentals," whereas the transitory component is thought to capture any remaining price changes or "noise." Hence, the real commodity price is assumed to evolve according to the following statistical process:

$$\breve{p}^*_{S,t} = a_t + \breve{b}_t, \quad \breve{b}_t = \rho \breve{b}_{t-1} + u_t, \quad a_t \sim N(0, \sigma_a^2), \quad u_t \sim N(0, \sigma_u^2), \tag{1}$$

for $t = 1, \ldots, T$, where $\breve{b}_t = \log(b_t/\bar{p}^*_S)$. Throughout, we assume that $\rho \in [0, 1)$. For $\rho = 1$, this model is known as the local level model (see Harvey, Koopman, and Shephard 2004). We impose the constraint that $\rho < 1$ to rule out non-stationary dynamics of the real commodity price. This assumption as well as the assumption of exogeneity of the commodity price implicit in (1) will be formally tested for the case of the copper price in section 3.5.

We assume that the effective real commodity price is fully observable to all agents but its individual components are not. Instead, following Erceg and Levin (2003), the agents infer the unobserved components in an optimal linear way using the Kalman filter, according to which $\widehat{\breve{b}}_t = \rho \widehat{\breve{b}}_{t-1} + K_t \rho^{-1}(\breve{p}^*_{S,t} - \rho \widehat{\breve{b}}_{t-1})$ and $\widehat{a}_t = \breve{p}^*_{S,t} - \widehat{\breve{b}}_t$. The variables $\widehat{a}_t$ and $\widehat{\breve{b}}_t$ denote the optimal linear inferences of a_t and $\breve{b}_t$ conditional on the information available at time t, i.e., $\widehat{a}_t = E[a_t|\mathcal{I}_t]$ and $\widehat{\breve{b}}_t = E[\breve{b}_t|\mathcal{I}_t]$, where $\mathcal{I}_t = \{p^*_{S,1}, p^*_{S,2}, p^*_{S,3}, \ldots, p^*_{S,t}\}$. Note that the current inference $\widehat{\breve{b}}_t$ and the forecasts $\widehat{\breve{b}}_{t+h} = \rho^h \widehat{\breve{b}}_t$ for horizon $h = 1, 2, 3, \ldots$ are adjusted at a rate $K_t \rho^{-1}$ with the prediction errors from the previous period's forecast, i.e., $\breve{p}^*_{S,t} - \rho \widehat{\breve{b}}_{t-1}$, where K_t is the Kalman gain parameter. For the estimation of the model, we will make the standard simplifying assumption that the Kalman gain parameter associated with (1) is constant, i.e., $K_t = K$ (see Erceg and Levin 2003; Céspedes and Soto 2007).[6] When $K = \rho$, we obtain $\breve{p}^*_{S,t} = \widehat{\breve{b}}_t$, as all variation in the effective price is attributed to the persistent component. Then there is only one shock and agents have perfect information (i.e., $\sigma_a = 0$ with signal-to-noise ratio $\sigma_u/\sigma_a \to \infty$). For $K < \rho$, the signal-to-noise ratio decreases

[6]This assumption is justified if the Kalman filter has reached a steady state, because $K_t \to K$ as t grows.

with K, so a lower K reflects a more important imperfect information problem (see appendix 1 for a derivation of the signal-to-noise ratio).

3.2 Estimation of the Unobserved-Components Model

In order to estimate the parameters of the model described by (1), including the key Kalman gain parameter that determines the speed at which agents learn about the persistence of commodity price fluctuations, we use quarterly data on the real spot price of copper for the period 1960:Q1–2016:Q3 as well as long-run expectations on future real copper prices. The latter are measured by the reference price of copper from section 2, which is available from 1999 onwards.[7] The reference price is the forecast of the average real annual spot price over the following ten years. The panel of experts whose forecasts determine the reference price usually meets in the third quarter of each year. Thus, denoting the reference price as $p_{S,t}^{ref,*}$, we have

$$\log(p_{S,t}^{ref,*}) = E_t \left\{ \frac{1}{40} \sum_{i=1}^{40} \log(p_{S,t+i}^{*}) \right\}. \tag{2}$$

Defining $\breve{p}_{S,t}^{ref,*} = \log(p_{S,t}^{ref,*}/\bar{p}_S^*)$, the reference price can be expressed in terms of the inferred persistent component $\widehat{\breve{b}}_t$ as follows:

$$\breve{p}_{S,t}^{ref,*} = (\rho^2/40)(1-\rho^{40})(1-\rho)^{-1}\widehat{\breve{b}}_t. \tag{3}$$

Hence, we augment (1) by (3) and estimate the parameters $\bar{p}_S^*$, ρ, σ_u, and K jointly. Given these parameters, the implied signal-to-noise ratio σ_u/σ_a and the implied value of σ_a is obtained from the Kalman filter recursions, as explained in detail in appendix 1.

[7]The reference price has the advantage that it is a forecast in real terms and it is therefore closely related to (1). A disadvantage is that the forecast is only reported once per year. An alternative approach could exploit quarterly nominal prices from futures contracts for copper to measure expectations. However, this approach would not only require augmenting (1) by a process for the underlying price deflator, but it would also require us to make additional assumptions to identify fluctuations in risk premia, so this approach would be more difficult to implement than our approach based on the reference price. Finally, we did not use the forecasts of professional forecasters from section 2, since those forecasts are available neither in real terms nor at a quarterly frequency.

To accommodate the unbalanced panel, we apply a two-step Bayesian data augmentation approach. In the first step, we use the data for the effective real copper price before 1999:Q3 (the first observation of the reference price) to estimate the parameters $\bar{p}_S^*$, ρ, and σ_u based on a simple AR(1) process by constrained maximum likelihood (ML). In the second step, we postulate informative priors for these parameters centered around the ML estimates from the first step and a diffuse prior for K for a Bayesian estimation using the data from 1999:Q3 onwards, now observing both the effective real spot price and the reference price of copper.[8]

The priors and posterior estimates are reported in table 1. The left panel displays the univariate prior and joint posterior distributions using as observed variables the reference price and the nominal spot price of refined copper (in dollars per metric pound) at the London Metal Exchange, which is deflated by an index of external prices for Chile that will also be used in the estimation of the DSGE model in section 4.8. The latter is constructed from an external price index published by the Central Bank of Chile ("IPE") from 1986:Q1 onwards, which is backcasted using the quarterly growth rate of the seasonally adjusted U.S. CPI for all urban consumers and all items. As a robustness exercise, the right panel of table 1 displays the prior and posterior densities when we use the U.S. CPI instead of the Chilean IPE as price deflator. In each case, we postulate a normal prior for $\log(\bar{p}_S^*)$, a beta prior for ρ, an inverse gamma prior for σ_u, and a uniform prior on the interval $[0, 1]$ for K (all assumed to be independent).

The estimated benchmark value of K when we use the Chilean IPE as price deflator is 0.17 and relatively well identified. The posterior mean implies a learning horizon of around four to five years in line with the observed revisions of the forecasts from figures 4 through 6. The estimates further imply an AR(1) parameter of the persistent price component of about 0.99, a quarterly standard deviation of innovations to the persistent component of approximately 8 percent, and an implied standard deviation of innovations to the transitory component of about 43 percent, such that the estimated

[8]The second step uses the Kalman filter for missing observations (these correspond to the missing observations of the reference price for the first, third, and fourth quarters of each year).

Table 1. Parameters of Unobserved-Components Model for the Real Price of Copper

		Using Chilean IPE as Deflator				Using U.S. CPI as Deflator			
		Prior Param.		Posterior		Prior Param.		Posterior	
Param.	**Prior Dist.**	**Mean**	**S.D.**	**Mean**	**90% HPDI**	**Mean**	**S.D.**	**Mean**	**90% HPDI**
$\text{Log}(\bar{p}_S^*)$	N	5.398	0.05	5.363	[5.283, 5.453]	5.350	0.05	5.296	[5.210, 5.374]
ρ	B	0.967	0.01	0.994	[0.991, 0.997]	0.963	0.01	0.991	[0.988, 0.995]
σ_u	IG	0.119	Inf.	0.083	[0.057, 0.107]	0.121	Inf.	0.089	[0.062, 0.116]
K	U	0.5	0.289	0.170	[0.116, 0.224]	0.5	0.289	0.181	[0.118, 0.234]
σ_a				0.430				0.426	

Data Source: Central Bank of Chile.
Notes: This table shows the priors (calibrated based on maximum-likelihood estimates for the sample 1960:Q1–1999:Q2) and posteriors (sample 1999:Q3–2016:Q3) with the 90 percent highest posterior density interval (HPDI) for the unobserved-components model (1) with (3), using as observables the real price of copper and the reference price of copper (from 1999:Q3 onwards) and as alternative deflators for the nominal copper price the Chilean external price index (IPE) and the U.S. CPI. In each case, the posterior distributions are computed from a Metropolis-Hastings chain of 30,000 draws with an average acceptance rate of approximately 25 percent, dropping the first 10,000 draws for convergence.

signal-to-noise ratio is around one-fifth. The results do not change much when we use the U.S. CPI as price deflator. The posterior mean of K increases only slightly to about 0.18, and the posterior means of the remaining parameters are also similar to the respective benchmark estimates. Hence, these estimates suggest an important role for imperfect information and learning to explain the observed forecast revisions.

3.3 Commodity Price Decomposition

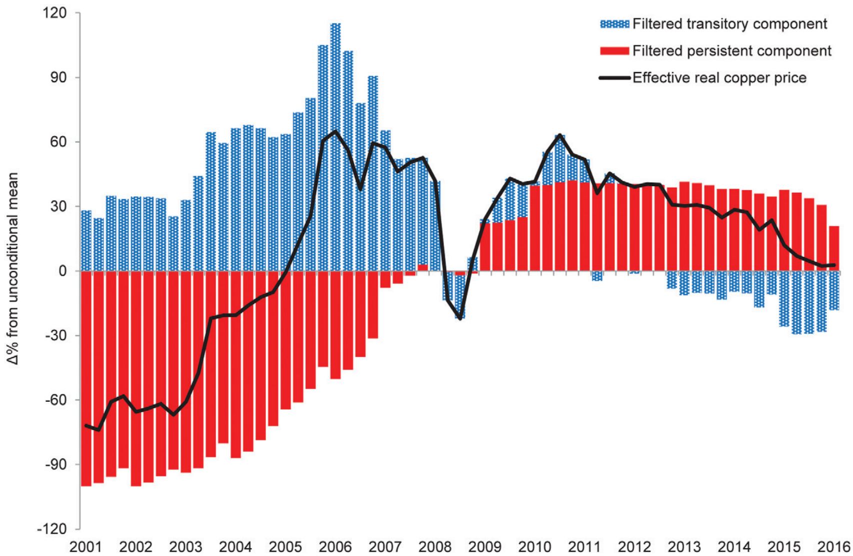
Figure 7 shows the Kalman-filter decomposition of the real copper price (deflated using the Chilean IPE and in log-deviations from its estimated unconditional mean) into its persistent and transitory components, which reflect the learning behavior by agents incorporated in our simple statistical model, over the period 2001:Q3–2016:Q3.[9] The persistent component starts out below the unconditional mean, reflecting the slump of commodity prices during the 1980s and 1990s. The increase in the copper price in the first half of the 2000s decade is mostly assigned to transitory shocks, while the contribution of the persistent component was still negative at that time. However, the persistent component gradually increases over time, as prices maintained their high levels during the second half of the 2000s decade and beyond, so that the price increases are gradually inferred to be more persistent, where the transitory component also captures most of the effect of the global financial crisis. Conversely, the recent decline is at first attributed to the transitory component and then to the persistent one.

3.4 Assessment against Alternative Models

A natural question is whether alternative statistical specifications for the commodity price, such as stationary autoregressive moving-average (ARMA) models or a non-stationary random-walk (RW) specification, fit the data on prices and forecasts better or worse than a model with persistent and transitory shocks such as ours. It may be the case that those alternative models can explain the

[9]The third quarter of 2001 is also the starting point of the sample that will be used for the estimation of the DSGE model in section 4.8.

Figure 7. Historical Decomposition of the Real Price of Copper, Unobserved-Components Model



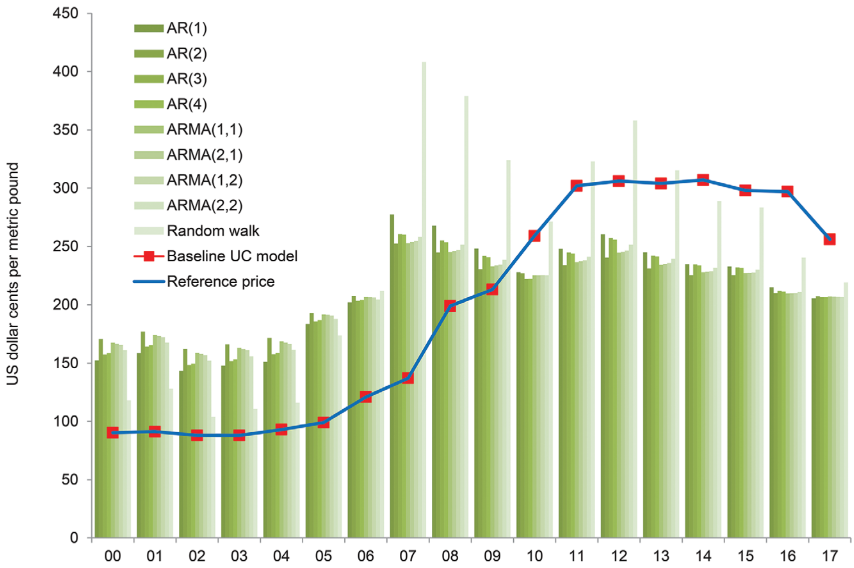
Notes: This figure shows the estimated decomposition of the quarterly log-deviations of the real price of copper from its estimated unconditional mean (sample 1999:Q3–2016:Q3) into a persistent component and a transitory component computed from the Kalman filter. The results are based on Bayesian estimates and the missing-observations Kalman filter as described in the text.

dynamics of both prices and forecasts in the same way or better than our model, but if that is not the case, this would provide support for our model and the learning mechanism it incorporates. To examine this question, we estimate various ARMA(p, q) models of the form

$$\breve{p}^*_{S,t} = \sum_{i=1}^{p} \phi_i \breve{p}^*_{S,t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j},$$

with different lag orders for the AR and MA parts (p and q) as well as a RW specification (i.e., $p = 1$, $q = 0$, and $\phi_1 = 1$). All models are augmented by (2) to compute the implied long-run forecasts, which can be compared with the actual forecasts (i.e., the reference price).

Figure 8. Fit of Reference Price, Unobserved-Components Model versus Alternative Models



Notes: This figure shows the predicted average real prices over the next ten years conditional on the data available in the third quarter of each year from alternative models against the actual reference price of copper. The autoregressive (AR) models, autoregressive moving-average (ARMA) models, and the random-walk (RW) model are estimated by maximum likelihood using the sample 1999:Q3–2016:Q3 for the real copper price deflated by the Chilean external price index IPE. The baseline unobserved-components (UC) model is estimated as explained in section 3.2.

All are estimated by maximum likelihood using data for 1999:Q3–2016:Q3 for the real copper price deflated by the Chilean external price index (IPE).[10]

The results are documented in figure 8, which plots the predicted average real prices over the next ten years conditional on the data available in the third quarter of each year against the actual reference price. The stationary ARMA models overpredict the observed forecasts in the first half of the sample and underpredict them in the

[10]Using a different deflator or a longer sample would not materially affect the results.

second half, the reason being the mean reversion inherent in these models. The RW model, on the other hand, exaggerates the effect of the initial price increases in the mid-2000s, which are immediately incorporated into the implied forecasts such that this model over-predicts the observed forecasts during that period. Our unobserved-components model, however, is capable of matching the observed forecasts due to the presence of persistent and transitory shocks. Hence, this exercise provides some support for our model. While alternative models with persistent and transitory shocks could be specified, a virtue of ours is that it is probably the most simple among this class of models. The key ingredients are imperfect information due to the presence of the two unobserved shocks and the backward-looking learning mechanism incorporated in our model. Any model incorporating these features is likely to generate similar results as ours.

3.5 Testing the Exogeneity and Stationarity Assumptions

Are the assumptions incorporated in (1) that the real copper price is stationary and exogenous for Chile appropriate? Here, we provide formal econometric tests of both assumptions. We first conduct augmented Dickey-Fuller (ADF) tests of the null hypothesis that the time series of the natural logarithm of the real copper price has a unit root. These tests are conducted using data for the relatively long time span 1960:Q1–2016:Q3 (225 observations). However, since the ADF test is known to have low power in finite samples under a large autoregressive root (see, e.g., DeJong et al. 1992), we further conduct some alternative tests proposed by Ng and Perron (2001). These tests are efficient versions of the modified Phillips-Perron (PP) test of Perron and Ng (1996) and have maximum power against very persistent alternatives.[11] All tests are again conducted using as alternative deflators for the nominal copper price the U.S. CPI or the Chilean external price index (IPE), which is again backcasted for the period before 1986:Q1 using the quarterly growth rates of the U.S.

[11]These tests also build upon the tests proposed by Bhargava (1986) and Elliott, Rothenberg, and Stock (1996). They do not exhibit the severe size distortions of the simple PP or ADF tests under large negative MA or AR roots, and they can have substantially higher power especially when the AR roots are close to unity.

CPI. The results are reported in table 2. Based on the ADF tests, the null hypothesis of a unit root is rejected at the 10 percent significance level, but barely so when using the IPE as deflator. However, based on the efficient modified PP tests, the null is rejected more decisively at lower significance levels of 5 percent or even 1 percent. We therefore conclude that the (log) real copper price is indeed a stationary series, and thus the assumption that $\rho < 1$ which was imposed through an informative prior in the Bayesian estimation of the unobserved-components model seems appropriate.

We now assess whether the real international copper price can indeed be taken as exogenous for Chile. In particular, we test the null hypothesis that the quarterly growth rate of real output in the Chilean copper sector does not cause the (log) real copper price in the sense of Granger (1969). According to data availability, these tests are conducted using the sample 1996:Q1–2016:Q3 (eighty-three observations). As before, two alternative price deflators are used as a robustness exercise. Table 3 shows the p-values of the associated Wald test for different lag lengths in the test equations. All p-values are higher than 0.1 such that in no case is the null rejected at the 10 percent significance level. Hence, we conclude from this exercise that the exogeneity assumption incorporated in (1) is appropriate for Chile, despite Chile being a major copper producer with a share of around one-third in global copper production.[12]

4. The DSGE Model

Our DSGE model is a quantitative New Keynesian small open-economy model for Chile based on Medina and Soto (2007, 2016). It includes a standard set of nominal and real rigidities (e.g., Christiano, Eichenbaum, and Evans 2005; Adolfson et al. 2007, 2008; Smets and Wouters 2007; Christoffel, Coenen, and Warne 2008), such as sticky prices and wages with partial indexation to past inflation, local currency pricing for imports and foreign currency pricing for exports, adjustment costs in investment, and habit persistence in consumption. It further includes non-Ricardian households and a structural balance fiscal rule to describe fiscal policy in Chile. On the

[12]Source: United States Geological Survey 2016, available at http://minerals.usgs.gov/minerals/pubs/mcs.

Table 2. Unit-Root Tests for the Real Price of Copper

	Using Chilean IPE as Deflator					Using U.S. CPI as Deflator				
		Efficient Modified PP					Efficient Modified PP			
	ADF	MZ_α	MZ_t	MSB	MP_T	**ADF**	MZ_α	MZ_t	MSB	MP_T
Test Statistic	−2.57*	−11.65**	−2.40**	0.21**	2.15**	−2.66*	−13.40**	−2.58***	0.19**	1.86**
1% Crit. Value	−3.46	−13.80	−2.58	0.17	1.78	−3.46	−13.80	−2.58	0.17	1.78
5% Crit. Value	−2.87	−8.10	−1.98	0.23	3.17	−2.87	−8.10	−1.98	0.23	3.17
10% Crit. Value	−2.57	−5.70	−1.62	0.28	4.45	−2.57	−5.70	−1.62	0.28	4.45

Data Source: Central Bank of Chile.
Notes: This table shows MacKinnon (1996) test statistics and critical values for the augmented Dickey-Fuller (ADF) test and Ng and Perron (2001) test statistics (MZ_α, MZ_t, MSB, and MP_T) and critical values for the efficient modified Phillips-Perron (PP) tests of the null hypothesis that the time series of the natural logarithm of the real copper price has a unit root, using as alternative deflators for the nominal copper price the Chilean external price index (IPE) and the U.S. CPI. The symbols *, **, and *** indicate rejection of the null hypothesis at the 10 percent, 5 percent, and 1 percent level, respectively. In each case, the sample used is 1960:Q1–2016:Q3 (225 observations) and the test equation includes a constant term and one lag, selected based on the Schwarz information criterion (using spectral GLS-detrended AR under the modified PP tests).

Table 3. Tests of Granger Causality from Chilean Copper Production to Real Price of Copper

	p-values of Wald Test	
Lag Length	**Using Chilean IPE as Deflator**	**Using U.S. CPI as Deflator**
1	0.340	0.261
2	0.308	0.189
3	0.586	0.394
4	0.615	0.479

Data Source: Central Bank of Chile.
Notes: This table shows the p-values for the Granger (1969) Wald test of the null hypothesis that the growth rate of real output in the Chilean copper sector (national accounts data, first differences of natural logarithms) does not Granger-cause the natural logarithm of the real copper price, using as alternative deflators for the nominal copper price the Chilean external price index (IPE) and the U.S. CPI, and for different lag lengths in the test equations. In each case, the sample used is 1996:Q1–2016:Q3 (eighty-three observations) and the test equations include a constant term.

production side, the model incorporates a domestic tradable goods sector, oil as an intermediate input, and a commodity sector.[13]

We add to this model two main features. First, the commodity sector expands as capital is allocated to that sector. Capital accumulation is subject to time to build and adjustment costs in investment, following Kydland and Prescott (1982) and Uribe and Yue (2006).[14] Second, we depart from the full-information rational expectations assumption by including imperfect information on the persistence of commodity price shocks and learning behavior by the agents, as

[13]Pieschacón (2012) also studies how commodity price shocks transmit differently in the cases of Mexico and Norway. Her model differs from the model of Medina and Soto (2007, 2016) in the following dimensions: (i) it adds nontradable goods, (ii) it includes public goods in the private consumption basket, (iii) it assumes only Ricardian agents, and (iv) it incorporates a different fiscal rule.

[14]While time to build as in Kydland and Prescott (1982) introduces lags between the initiation of investment projects and effective production, it tends to imply "jumpy" dynamics of investment that are inconsistent with the data. We therefore allow for adjustment costs in investment projects in addition to time to build, following Uribe and Yue (2006), to better match investment dynamics.

described in section 3. In order to focus on these extensions, the description of the remaining model structure is deliberately brief and we refer to Medina and Soto (2007) for details.

4.1 Households

There is a continuum of households indexed by $j \in [0, 1]$. A fraction λ of those households are non-Ricardian (hand-to-mouth) households that do not have access to the capital market. The remaining, Ricardian households do have access to the capital market and thus make intertemporal consumption and savings decisions in a forward-looking manner.

Households of the Ricardian type maximize the present value of expected utility at time t:

$$\max E_t \sum_{i=0}^{\infty} \beta^i \zeta_{C,t+i} \left[\log(C^R_{t+i}(j) - hC^R_{t+i-1}) - \psi \frac{l_{t+i}(j)^{1+\sigma_L}}{1+\sigma_L} \right],$$
$$j \in (1-\lambda, 1],$$

subject to the period-by-period budget constraint

$$P_{C,t}C^R_t(j) + E_t\{d_{t,t+1}D_{t+1}(j)\} + \frac{B_t(j)}{r_t} + \frac{\varepsilon_t B^*_{P,t}(j)}{r^*_t \Theta_t}$$
$$= W_t(j)l_t(j) + \Xi_t(j) - TAXN_t(j) + D_t(j)$$
$$+ B_{t-1}(j) + \varepsilon_t B^*_{P,t-1}(j),$$

where $C^R_t(j)$ is consumption of household j and C^R_t is aggregate consumption of Ricardian households, respectively; $l_t(j)$ is household j's labor effort (in hours); and $\zeta_{C,t}$ is a preference shock to the households' discount factor. Further, $P_{C,t}$ is the aggregate consumer price index (CPI); $W_t(j)$ is the nominal wage set by household j; $\Xi_t(j)$ collects payouts by firms; $TAXN_t(j)$ are lump-sum tax payments to the government; ε_t is the nominal exchange rate (units of domestic currency per unit of foreign currency); and $d_{t,t+1}$ is the period-t price of one-period domestic contingent bonds, $D_t(j)$, normalized by the probability of the occurrence of the state. The variable r_t denotes the gross interest rate on a non-contingent domestic bond denominated in domestic currency, $B_t(j)$, whereas r^*_t is the interest rate on a non-contingent foreign bond denominated in foreign currency, $B^*_{P,t}(j)$. The term $\Theta(\cdot)$ is a premium paid by domestic agents on top of the foreign interest rate.[15]

Following Erceg, Henderson, and Levin (2000), each household is a monopolistic supplier of a differentiated labor service. These labor services are bundled by a set of perfectly competitive labor packers and combined into an aggregate labor service unit used as an input in production of domestic intermediate varieties. Cost minimization of labor packers yields the demand for each type of labor and aggregate labor demand by firms. There are wage rigidities in the spirit of Calvo (1983). Each period, a randomly selected fraction $(1-\phi_L)$ of households is able to reoptimize their nominal wage. The reoptimizing households maximize the expected discounted future stream of labor income net of the disutility from work, subject to the labor demand constraint. All those that cannot reoptimize set their wages according to a weighted average of past CPI inflation and the inflation target set by the central bank. Once a household has set its wage, it must supply any quantity of labor service demanded at that wage.

Households of the non-Ricardian type consume their disposable wage income each period:

$$P_{C,t}C_t^{NR}(j) = W_t l_t(j) - TAXN_t(j), \qquad j \in [0, \lambda].$$

For simplicity, it is assumed that non-Ricardian households set a wage equivalent to the average wage set by Ricardian households. As a consequence, the supply of labor by non-Ricardian households coincides with the average labor supply of Ricardian households.

The households' consumption bundle is a constant elasticity of substitution (CES) composite of a core consumption bundle, $C_{Z,t}(j)$, and oil consumption, $C_{O,t}(j)$. Core consumption is a CES composite of final domestic goods, $C_{H,t}(j)$, and imported goods, $C_{F,t}(j)$. Households minimize the costs of the different bundles, which yields

[15] The premium is a function of the aggregate (private, $B^*_{P,t}$, plus government, $B^*_{G,t}$) net foreign bond position relative to nominal GDP ($BY_t = \varepsilon_t B^*_t / P_{Y,t} Y_t$), i.e., $\Theta_t = \bar{\Theta} \exp[-\varrho(BY_t - \overline{BY}) + \zeta_{\Theta,t}/\bar{\zeta}_{\Theta} - 1]$, where $\varrho > 0$ to ensure stationarity of net foreign bonds and where $\zeta_{\Theta,t}$ is a shock to the premium (throughout, bars indicate deterministic steady-state values).

standard Dixit-Stiglitz type demand functions for each component and expressions for the headline CPI and the core CPI excluding oil.

4.2 Domestic Goods

In the domestic goods sector, there is a continuum of firms that produce differentiated varieties of intermediate tradable goods using labor, capital, and oil as inputs. They have monopoly power over the varieties they produce and adjust prices subject to Calvo frictions. These firms sell their varieties to competitive assemblers that produce final domestic goods that are sold in the domestic and foreign markets. Another set of competitive firms produces the capital goods used in intermediate goods production. All firms in this sector are owned by Ricardian households.

A representative capital goods producer rents out capital goods to domestic intermediate goods producers. It decides how much capital to accumulate each period, assembling investment goods, I_t, with a CES technology that combines final domestic goods, $I_{H,t}$, and imported goods, $I_{F,t}$. The optimal composition of investment is determined through cost minimization. The firm may adjust investment to produce new capital goods, K_t, in each period but there are convex costs of adjusting investment, $\Phi(\cdot)$, following Christiano, Eichenbaum, and Evans (2005). The firm maximizes the present value to households of expected profits:

$$\max E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}(Z_{t+i}K_{t+i-1} - P_{I,t+i}I_{t+i}),$$

subject to the law of motion of capital $K_t = (1-\delta)\,K_{t-1} + [1 - \Phi(I_t/I_{t-1})]\zeta_{I,t}I_t$, where $\Lambda_{t,t+i}$ is the stochastic discount factor for nominal payoffs of the Ricardian households and Z_t is the rental price of capital.[16] The variable $\zeta_{I,t}$ is an investment-specific shock that alters the rate at which investment is transformed into capital (see Greenwood, Hercowitz, and Krusell 2000).

[16] The stochastic discount factor satisfies $\Lambda_{t,t+i} = \beta^i(\zeta_{C,t+i}/\zeta_{C,t})(C_t^R - hC_{t-1}^R)/(C_{t+i}^R - hC_{t+i-1}^R)(P_{C,t}/P_{C,t+i})$. Further, the function $\Phi(\cdot)$ satisfies $\Phi(1+g_Y) = \Phi'(1+g_Y) = 0$ and $\Phi''(1+g_Y) = \mu_S > 0$, where g_Y is the steady-state (balanced) growth rate of the economy.

There is a large set of firms that use a CES technology to assemble final domestic goods from domestic intermediate varieties. A quantity $Y_{H,t}$ of those goods is sold domestically and a quantity $Y^*_{H,t}$ is sold abroad. The assemblers demand intermediate goods of variety z_H for domestic sale, $Y_{H,t}(z_H)$, and intermediate goods for foreign sale, $Y^*_{H,t}(z_H)$. Input cost minimization yields the typical Dixit-Stiglitz demand functions for each variety.

Intermediate goods producers decide on the most efficient combination of labor, capital, and oil to minimize input costs given factor prices. Their technology is as follows:

$$\mathbf{Y}_{H,t}(z_H) = a_{H,t}\left[{\alpha_H}^{1/\omega_H} V_{H,t}(z_H)^{1-1/\omega_H} + (1-\alpha_H)^{1/\omega_H} O_{H,t}(z_H)^{1-1/\omega_H}\right]^{\frac{\omega_H}{\omega_H - 1}},$$

where $V_{H,t}$ denotes value added produced out of labor and capital, while $O_{H,t}(z_H)$ is the amount of oil used as intermediate input, and $a_{H,t}$ represents a stationary productivity shock common to all firms. Value added is generated through a Cobb-Douglas function: $V_{H,t}(z_H) = [T_t l_t(z_H)]^{\eta_H} K_{t-1}(z_H)^{1-\eta_H}$, where $l_t(z_H)$ denotes the labor utilized, T_t is a stochastic trend in labor productivity, and $K_{t-1}(z_H)$ is the capital rented at the beginning of period t.[17]

The intermediate goods producers have monopoly power and set their prices separately in the domestic market, $P_{H,t}(z_H)$, and the foreign market, $P^*_{H,t}(z_H)$, maximizing profits subject to the corresponding demand constraints. Prices are set in a staggered way. Each period, the probability that a firm receives a signal for optimally adjusting its price for the domestic market is $1 - \phi_{H_D}$, while the probability of optimally adjusting its price for the foreign market is $1 - \phi_{H_F}$. If a firm does not receive a signal, it updates its price according to a weighted average of past changes of aggregate producer prices ($P_{H,t}$ and $P^*_{H,t}$ for the respective markets) and their steady-state inflation rates. Let $MC_{H,t}$ denote the marginal cost of producing variety z_H. When a firm can reoptimize its price for the domestic market, it solves

[17]The productivity trend evolves according to the process $T_t/T_{t-1} = \zeta_{T,t} = (1+g_Y)^{1-\rho_T}\zeta^{\rho_T}_{T,t-1}\exp(\varepsilon_{T,t})$.

$$\max E_t \sum_{i=0}^{\infty} \phi^i_{H_D} \Lambda_{t,t+i} [\Gamma^i_{H_D,t} P_{H,t}(z_H) - MC_{H,t+i}] Y_{H,t+i}(z_H),$$

subject to the final domestic goods producer's demand $Y_{H,t}(z_H) = [P_{H,t}(z_H)/P_{H,t}]^{-\epsilon_H} Y_{H,t}$. Analogously, when a firm can reoptimize its price for the foreign market, it solves

$$\max E_t \sum_{i=0}^{\infty} \phi^i_{H_F} \Lambda_{t,t+i} [\Gamma^i_{H_F,t} \varepsilon_{t+i} P^*_{H,t}(z_H) - MC_{H,t+i}] Y^*_{H,t+i}(z_H),$$

subject to the demand constraint $Y^*_{H,t}(z_H) = [P^*_{H,t}(z_H)/P^*_{H,t}]^{-\epsilon_H} Y^*_{H,t}$.[18]

4.3 Imported Goods

In the imported goods sector, there is a continuum of retail firms that repackage a homogeneous good bought from abroad into differentiated imported varieties through a brand-naming technology. Those firms have monopoly power in the domestic retailing of their particular variety and set prices subject to Calvo frictions. They sell their varieties to competitive assemblers that produce final imported goods that are bought by households and firms. All firms in the imported goods sector are owned by Ricardian households.

There is a large set of firms that use a CES technology to assemble final imported goods, $Y_{F,t}$, from imported varieties. Demand for a particular imported variety, $Y_{F,t}(z_F)$, is determined through cost minimization, which yields the Dixit-Stiglitz demand functions for each variety.

Imported goods retailers buy a homogeneous good from abroad at the price $P^*_{F,t}$, which is then differentiated into a particular variety and sold domestically to assemblers of final imported goods. It takes one unit of the homogenous foreign good to produce a unit of retail output. Each importing firm has monopoly power and adjusts the domestic price of its variety in a staggered way. Each period, a firm

[18]For $i > 1$, the passive price updating rules are $\Gamma^i_{H_D,t} = \Gamma^{i-1}_{H_D,t} \pi^{\chi_{H_D}}_{H,t+i-1} \bar{\pi}^{1-\chi_{H_D}}$ for domestically sold goods and $\Gamma^i_{H_F,t} = \Gamma^{i-1}_{H_F,t} \pi^{*\chi_{H_F}}_{H,t+i-1} \bar{\pi}^{*1-\chi_{H_F}}$ for goods sold abroad, where $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, $\pi^*_{H,t} = P^*_{H,t}/P^*_{H,t-1}$, and $\bar{\pi}^*$ denotes steady-state foreign CPI inflation. For $i = 0$, we have $\Gamma^0_{H_D,t} = \Gamma^0_{H_F,t} = 1$.

optimally adjusts its price with probability $1-\phi_F$. If a firm does not receive a signal, it updates its price according to a weighted average of past changes of aggregate producer prices ($P_{F,t}$) and steady-state inflation. A reoptimizing firm solves

$$\max E_t \sum_{i=0}^{\infty} \phi_F^i \Lambda_{t,t+i}[\Gamma_{F,t}^i P_{F,t}(z_F) - \varepsilon_{t+i} P_{F,t+i}^*] Y_{F,t+i}(z_F),$$

subject to the final imported goods producer's demand $Y_{F,t}(z_H) = [P_{F,t}(z_F)/P_{F,t}]^{-\epsilon_F} \mathrm{Y}_{F,t}$.[19]

4.4 Commodity Goods

We extend the model of Medina and Soto (2007) by endogenizing commodity production. There is a representative firm in the commodity sector (S) that produces a homogeneous commodity good. The entire production is exported. A fraction χ of the firm's assets is owned by the government and the remaining fraction is owned by foreign investors. The firm's revenue is shared accordingly, but the government levies taxes on the profits that accrue to foreign investors.

The firm uses capital specific to the sector, $K_{S,t}$, to produce commodities, $Y_{S,t}$. Production evolves along the balanced growth path of the economy, but there may be transitory deviations from that path due to sectoral technology shocks, $a_{S,t}$. Specifically, production satisfies

$$Y_{S,t} = a_{S,t} F^S(T_t, K_{S,t-1}). \tag{4}$$

The function $F^S(\cdot)$ is homogeneous of degree one in its arguments and has diminishing returns to capital additions. While we focus on capital-intensive commodity production and abstract from other inputs such as labor and land for simplicity, the shock $a_{S,t}$ could be interpreted to capture any variations in such additional inputs.[20]

[19]The passive price updating rule is $\Gamma_{F,t}^i = \Gamma_{F,t}^{i-1} \pi_{F,t+i-1}^{\chi_F} \bar{\pi}^{1-\chi_F}$ for $i > 1$ and $\Gamma_{F,t}^0 = 1$ for $i = 1$.

[20]For instance, we could take the Cobb-Douglas production function $Y_{S,t} = \tilde{F}^S(T_t l_S, T_t \digamma_{S,t}, K_{S,t-1}) = (T_t l_S)^{\eta_{l_S}} (T_t \digamma_{S,t})^{\eta_{\digamma_S}} K_{S,t-1}^{1-\eta_{l_S}-\eta_{\digamma_S}}$, where l_S would

We also allow for a fixed transfer to households to capture eventual labor remunerations or other fixed costs (see below).

Let $P^*_{S,t}$ denote the international price of the commodity good and let $P_{S,t} = \varepsilon_t P^*_{S,t}$ be its domestic price, which the firm takes as given. Gross profits of the firm are given by $\Pi_{S,t} = P_{S,t}Y_{S,t} - P_{C,t}T_t\kappa_S$, where $P_{C,t}T_t\kappa_S$ is a fixed cost of production that grows at the same rate as nominal output. We assume that this fixed cost is a lump-sum transfer to Ricardian households. The cash flow of the firm is $CF_{S,t} = \Pi_{S,t} - P_{I_S,t}I_{S,t}$, where $P_{I_S,t}I_{S,t}$ is the firm's investment. The objective of the firm is to maximize the present real value of its expected cash flow:

$$\max E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}(S)\frac{CF_{S,t+i}}{P_{C,t+i}},$$

where $\Lambda_{t,t+i}(S)$ denotes the stochastic discount factor relevant to the firm. This discount factor is taken to be identical to the one of the Ricardian households, i.e., $\Lambda_{t,t+i}(S) = \Lambda_{t,t+i}$.[21]

The stock of capital in sector S is augmented through investment projects, $X_{S,t}$. Following Uribe and Yue (2006), there are adjustment costs in investment and time to build in the installation of capital à la Kydland and Prescott (1982). In particular, the firm can start new investment projects in each period but at a convex cost: the larger the change in investment, the larger the implied cost. In addition, new investment projects take $n \geq 1$ periods to mature. Collecting these assumptions results in the following law of motion of capital:

$$K_{S,t} = (1-\delta_S)K_{S,t-1} + [1 - \Phi_S(X_{S,t-n+1}/X_{S,t-n})]X_{S,t-n+1}. \quad (5)$$

The function $\Phi_S(\cdot)$ is analogous to the Christiano, Eichenbaum, and Evans (2005) style flow adjustment cost function from the law of

be a fixed input of labor and $\digamma_{S,t}$ would capture variations in other factors such as the mineral content of land. Defining $\eta_S = \eta_{l_S} + \eta_{\digamma_S}$, we obtain $Y_{S,t} = a_{S,t}T_t^{\eta_S}K_{S,t-1}^{1-\eta_S}$, which is a representation of (4) with $a_{S,t} = l_S^{\eta_{l_S}}\digamma_{S,t}^{\eta_{\digamma_S}}$ and with $F^S(T_t, K_{S,t-1}) = T_t^{\eta_S}K_{S,t-1}^{1-\eta_S}$. Under those assumptions, total factor productivity $a_{S,t}$ is a function of labor and other factors subsumed in $\digamma_{S,t}$.

[21]The relation $\Lambda_{t,t+i}(S) = \Lambda_{t,t+i}$ holds, as we assume, if the government has a stochastic discount factor equivalent to that of the households and if foreign investors have access to domestic currency bonds.

motion of capital used in the domestic goods sector, and satisfies $\Phi_S(1+g_Y) = \Phi'_S(1+g_Y) = 0$ and $\Phi'_S(1+g_Y) = \mu_{I_S} > 0$. A similar specification of the law of motion of capital is employed in Uribe and Yue (2006). The effective flow of investment in period t is given by

$$I_{S,t} = \sum_{j=0}^{n-1} \varphi_j X_{S,t-j}, \tag{6}$$

where φ_j denotes the fraction of projects initiated in period $t-j$ that is financed in period t, with $\sum_{j=0}^{n-1} \varphi_j = 1$. We will assume that $\varphi_0 = \varphi_1 = \ldots = \varphi_{n-1}$, as in Kydland and Prescott (1982), i.e., the cost of a project is spread equally over the horizon of its installation. In the extreme when $n = 1$, we obtain the familiar law of motion $K_{S,t} = (1-\delta_S)K_{S,t-1} + [1 - \Phi_S(I_{S,t}/I_{S,t-1})]I_{S,t}$.

The firm's first-order optimality conditions are as follows:

$$K_{S,t} : \frac{Q_{S,t}}{P_{C,t}} = E_t\left\{\Lambda_{t,t+1}\left[\frac{Q_{S,t+1}}{P_{C,t+1}}(1-\delta_S) + \frac{P_{S,t+1}a_{S,t+1}F^S_{K_S}(T_{t+1}, K_{S,t})}{P_{C,t+1}}\right]\right\},$$

$$X_{S,t} : \varphi_0 \frac{P_{I_S,t}}{P_{C,t}} + \varphi_1 E_t\left\{\Lambda_{t,t+1}\frac{P_{I_S,t+1}}{P_{C,t+1}}\right\} + \cdots + \varphi_{n-1}E_t\left\{\Lambda_{t,t+n-1}\frac{P_{I_S,t+n-1}}{P_{C,t+n-1}}\right\}$$
$$= E_t\left\{\begin{array}{c} \Lambda_{t,t+n-1}\frac{Q_{S,t+n-1}}{P_{C,t+n-1}}[1 - \Phi_S(X_{S,t}/X_{S,t-1}) \\ -\Phi'_S(X_{S,t}/X_{S,t-1})X_{S,t}/X_{S,t-1}] \\ +\Lambda_{t,t+n}\frac{Q_{S,t+n}}{P_{C,t+n}}\Phi'_S(X_{S,t+1}/X_{S,t})(X_{S,t+1}/X_{S,t})^2 \end{array}\right\},$$

where $F^S_{K_S}(\cdot)$ is the derivative of the production function in (4) with respect to capital. These two conditions jointly determine the evolution of investment projects and the mark-to-market value of capital, $Q_{S,t}$, in sector S. The law of motion (5) determines the evolution of the stock of capital and (6) determines the effective flow of investment in this sector.

The investment good that is required to build the stock of capital in sector S is a CES bundle of final domestic goods, $I_{H,t}(S)$, and imported goods, $I_{F,t}(S)$:

$$I_{S,t} = \left[\gamma_{I_S}^{1/\eta_{I_S}} I_{H,t}(S)^{1-1/\eta_{I_S}} + (1-\gamma_{I_S})^{1/\eta_{I_S}} I_{F,t}(S)^{1-1/\eta_{I_S}}\right]^{\frac{\eta_{I_S}}{\eta_{I_S}-1}}. \tag{7}$$

The optimal composition of investment is determined through cost minimization. Each period, given the effective flow of investment, the firm minimizes $P_{I_S,t}I_{S,t} = P_{H,t}I_{H,t}(S) + P_{F,t}I_{F,t}(S)$ subject to (7), which yields the demands for investment inputs originating in sector S: $I_{H,t}(S) = \gamma_{I_S}(P_{H,t}/P_{I_S,t})^{-\eta_{I_S}} I_{S,t}$ and $I_{F,t}(S) = (1-\gamma_{I_S})(P_{F,t}/P_{I_S,t})^{-\eta_{I_S}} I_{S,t}$.

4.5 Fiscal and Monetary Policy

A share χ of the cash flow generated in sector S goes directly to the government, and the government also levies taxes at a fixed rate τ_S on the profits—net of depreciation—that accrue to foreign investors. The budget constraint of the government is therefore as follows:

$$\begin{aligned} P_{G,t}G_t + \frac{\varepsilon_t B^*_{G,t}}{r^*_t \Theta_t} &= \varepsilon_t B^*_{G,t-1} + \tau_t P_{Y,t}Y_t + \chi CF_{S,t} \\ &\quad + \tau_S(1-\chi)(\Pi_{S,t} - \delta_S Q_{S,t}K_{S,t-1}), \end{aligned}$$

where $P_{G,t}G_t$ denotes nominal government consumption expenditure, $B^*_{G,t}$ is the government net foreign asset position, and τ_t are lump-sum taxes from households net of transfers (as a share of nominal GDP, $P_{Y,t}Y_t$). Note that the government net asset position is assumed to be completely denominated in foreign currency, as in Medina and Soto (2007). Government consumption is characterized by complete home bias, i.e., $G_t = G_{H,t}$ and $P_{G,t} = P_{H,t}$.

Government expenditure follows a structural balance fiscal rule analogous to the one described in Medina and Soto (2007):

$$\begin{aligned} \frac{P_{G,t}G_t}{P_{Y,t}Y_t} &= \left[\begin{array}{c} \left(1 - \frac{1}{r^*_{t-1}\Theta_{t-1}}\right)\frac{\varepsilon_t B^*_{G,t-1}}{P_{Y,t}Y_t} + \tau_t \frac{P_{Y,t}\bar{Y}_t}{P_{Y,t}Y_t} + \chi\frac{CF_{S,t}}{P_{Y,t}Y_t} \\ +\tau_S(1-\chi)\frac{\Pi_{S,t}-\delta_S Q_{S,t}K_{S,t-1}}{P_{Y,t}Y_t} - \frac{VC_t}{P_{Y,t}Y_t} - \bar{s}_B \end{array}\right] \\ &\quad \times \frac{P_{G,t}\zeta_{G,t}T_t}{P_{Y,t}Y_t}, \end{aligned}$$

where $VC_t = [\chi + \tau_S(1-\chi)]\varepsilon_t(P^*_t - \tilde{P}^*_t)Y_{S,t}$ is the cyclical adjustment of the rule that depends crucially on the difference between the effective commodity price, P^*_t, and the long-run reference price,

$\tilde{P}_t^*$, which is calculated as the forecast of the effective commodity price averaged over a ten-year horizon. In addition, $\bar{Y}_t$ stands for potential real GDP, which for simplicity is taken to be equal to steady-state output, and the parameter $\bar{s}_B$ is the structural balance target. The variable $\zeta_{G,t}$ is a shock capturing deviations of government expenditure from the fiscal rule.

Monetary policy is conducted through a simple Taylor-type feedback rule for the nominal interest rate, which is a slightly modified version of the one specified in Medina and Soto (2007). In particular, while the latter assume that the central bank responds entirely to deviations of core CPI inflation from target and of output growth from potential growth, we allow for a partial response to headline CPI inflation to capture possible concerns by the central bank on non-core inflation. Hence, the monetary policy rule is specified as follows:

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\psi_r} \left[\left(\frac{\pi_{Z,t}}{\bar{\pi}}\right)^{\psi_\pi \psi_{\pi Z}} \left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_\pi (1-\psi_{\pi Z})} \left(\frac{Y_t/Y_{t-1}}{T_t/T_{t-1}}\right)^{\psi_Y}\right]^{1-\psi_r} \times \exp(\zeta_{m,t}),$$

where $\pi_{Z,t}$ and π_t are core and headline CPI inflation, respectively, Y_t is real GDP, and $\zeta_{m,t}$ is an iid shock that captures deviations of the interest rate from the monetary policy rule.

4.6 Rest of the World

Foreign agents demand the commodity good and the final domestic good. They supply oil and the homogeneous good that is bought by importing firms. Foreign demand for the commodity good is assumed to be completely elastic at its international price, $P_{S,t}^*$. Likewise, foreign supply of oil is assumed to be completely elastic at any given price, $P_{O,t}^*$. The real exchange rate is defined as the domestic currency price of a foreign price index, $\varepsilon_t P_t^*$, relative to the domestic CPI. The domestic economy is assumed to be small relative to the rest of the world. As a consequence, the price of the homogeneous foreign good, $P_{F,t}^*$, coincides with the foreign price index.[22] Foreign demand for the final domestic good depends on its relative price

[22]We drop a shock to the ratio $P_{F,t}^*/P_t^*$ that is present in the model of Medina and Soto (2007) because this shock was not identified in our estimations.

abroad, $P^*_{H,t}/P^*_t$, and foreign aggregate demand, Y^*_t, according to the demand function $Y^*_{H,t} = \zeta^*(P^*_{H,t}/P^*_t)^{-\eta^*}Y^*_t$.

4.7 Aggregate Equilibrium

The market clearing condition for each variety of domestic goods is $\mathbf{Y}_{H,t}(z_H) = Y_{H,t}(z_H) + Y^*_{H,t}(z_H)$, where $Y_{H,t}(z_H) = [P_{H,t}(z_H)/P_{H,t}]^{-\epsilon_H}Y_{H,t}$ and $Y^*_{H,t}(z_H) = [P^*_{H,t}(z_H)/P^*_{H,t}]^{-\epsilon_H}Y^*_{H,t}$, and where $Y_{H,t} = C_{H,t} + I_{H,t} + I_{H,t}(S) + G_{H,t}$. In the labor market, labor demand by intermediate goods producers equals labor supply: $\int_0^1 l_t(z_H)dz_H = l_t$, where the aggregate labor service unit is given by $l_t = [\int_0^1 l_t(j)^{1-1/\varepsilon_L}dj]^{(\varepsilon_L-1)/\varepsilon_L}$. Nominal GDP satisfies $P_{Y,t}Y_t = P_{C,t}C_t + P_{I,t}I_t + P_{I_S,t}I_{S,t} + P_{G,t}G_t + P_{X,t}X_t - P_{M,t}M_t$, where $P_{X,t}X_t = \varepsilon_t(P^*_{H,t}Y^*_{H,t} + P^*_{S,t}Y_{S,t})$ and $P_{M,t}M_t = \varepsilon_t[P^*_{F,t}Y_{F,t} + P^*_{O,t}(C_{O,t} + O_{H,t})]$ are nominal exports and imports, respectively, with $Y_{F,t} = C_{F,t} + I_{F,t} + I_{F,t}(S)$. Real GDP is defined as $Y_t = C_t + I_t + I_{S,t} + G_t + X_t - M_t$. Substituting out aggregate profits in the budget constraint of the households and combining the latter with the budget constraint of the government yields the following expression for the evolution of aggregate net foreign bonds:

$$\frac{\varepsilon_t B^*_t}{r^*_t\Theta_t} = P_{X,t}X_t - P_{M,t}M_t + \varepsilon_t B^*_{t-1} - (1-\chi)CF_{S,t} + \tau_S(1-\chi)(\Pi_{S,t} - \delta_S Q_{S,t}K_{S,t-1}).$$

The terms on the right-hand side are net exports, net interest receipts minus the cash flow from the commodity sector that accrues to foreign investors, and transfers from foreigners due to taxes on profits net of the mark-to-market value of capital depreciation in the commodity sector. Finally, the current account balance is equivalent to the quarter-on-quarter change in the international investment position of the country (relative to nominal GDP):

$$CAY_t = \frac{1}{P_{Y,t}Y_t}\left[\frac{\varepsilon_t B^*_t}{r^*_t\Theta_t} - \frac{\varepsilon_t B^*_{t-1}}{r^*_{t-1}\Theta_{t-1}}\right] - (1-\chi)\frac{Q_{S,t}(K_{S,t} - K_{S,t-1})}{P_{Y,t}Y_t}.$$

Apart from the exogenous process of the commodity price described previously, there are twelve additional shocks in the model: preferences ($\zeta_{C,t}$), neutral technology ($a_{H,t}$), productivity growth ($\zeta_{T,t}$), investment-specific technology ($\zeta_{I,t}$), commodity-specific technology ($a_{S,t}$), fiscal policy ($\zeta_{G,t}$), monetary policy ($\zeta_{m,t}$), stationary foreign demand ($y_t^* = Y_t^*/T_t$), foreign inflation ($\pi_t^* = P_t^*/P_{t-1}^*$), the real oil price ($p_{O,t}^* = P_{O,t}^*/P_t^*$), the foreign interest rate (r_t^*), and the country premium ($\zeta_{\Theta,t}$). All of those shocks are assumed to follow mean-reverting AR(1) processes in logs, except the monetary policy shock, which is a zero-mean iid process in levels.

4.8 Estimation

The model is solved by a log-linear approximation around the deterministic steady state. The detailed description of the solution under learning is provided in appendix 2. The model is then estimated by Bayesian methods, as described in An and Schorfheide (2007). Here we briefly describe the estimation strategy and how we incorporate imperfect information in the estimation, before discussing the details of the procedure (data, priors, etc.).

Let x_t denote the model variables and let Y_t be a vector of observed variables of the econometrician. The log-linear solution of the model has the following state-space representation:

$$Y_t = Hx_t + v_t, \qquad x_t = D\widehat{a}_t + E\widehat{\breve{b}}_t + Fx_{t-1} + G\varepsilon_t,$$
$$v_t \sim N(0, \Sigma_v), \qquad \varepsilon_t \sim N(0, \Sigma_\varepsilon),$$

for $t = 1, \ldots, T$. The variables $\widehat{a}_t$ and $\widehat{\breve{b}}_t$ are defined as in section 3. The vector ε_t represents the innovations to the remaining exogenous processes and v_t is a vector of observation errors. Further, let θ collect the parameters of the model, let $P(\theta)$ be a prior density on those parameters, and let $L(Y^T|\theta)$ denote the likelihood function for the observed data, $Y^T = [Y_1, \ldots, Y_T]'$. The Kalman filter is applied to evaluate $L(Y^T|\theta)$, and the posterior density, $P(\theta|Y^T) = L(Y^T|\theta) \times P(\theta)/\int L(Y^T|\theta)P(\theta)d\theta$, is evaluated using the Metropolis-Hastings algorithm.[23]

We use quarterly data for Chile covering the period 2001:Q3–2015:Q4 for the estimation of the model.[24] The observed variables

[23]We use version 4.3.3 of the Dynare toolbox for MATLAB in the computations.

[24]While the Central Bank of Chile switched from partial to full inflation targeting under a flexible exchange rate already at the end of 1999, it has used an overnight interest rate as policy instrument only since July 2001, which we therefore use as the starting point of our sample.

include real GDP (Y_t), real copper mining output as a proxy of commodity production ($Y_{S,t}$), private non-durable consumption (C_t), total investment ($I_t + I_{S,t}$), government consumption (G_t), consumer price inflation ($\pi_{C,t}$), the short-term central bank target rate (r_t), the real exchange rate (rer_t), the current account balance as a ratio of nominal GDP (CAY_t), an indicator of export-weighted real GDP of Chile's main trading partners as a proxy for foreign demand ($Y_{F,t}$), the inflation rate of the external price index as a proxy for foreign inflation (π_t^*), the London Interbank Offered Rate as a proxy for the foreign interest rate (r_t^*), the J.P. Morgan Emerging Market Bond Index Global (EMBIG) spread for Chile as a proxy for the country premium (Θ_t), the price of West Texas Intermediate crude oil in dollars per barrel deflated by the external price index ($P_{O,t}^*/P_t^*$), and the London Metal Exchange price of refined copper in dollars per metric pound deflated by the external price index ($P_{S,t}^*/P_t^*$).[25] We further treat the inferred persistent component of the real copper price computed through the Kalman filter ($\widehat{\breve{b}}_t$) from section 3.3 as an observed variable.[26]

The calibrated parameters include some steady-state values, other parameters that are chosen to match sample averages or long-run ratios for the Chilean economy, as well as a number of parameters that were not identified or only weakly identified. In addition, we also fix the parameters of the observable exogenous processes prior to the actual estimation of the model. Table 4 presents the list of calibrated parameters.

We assume a steady-state labor productivity growth rate (g_Y) of 2 percent on an annual basis, which is consistent with an observed average real GDP growth rate of approximately 4 percent over the

[25]The data were obtained from the Central Bank of Chile at https://si3.bcentral.cl/Siete/secure/cuadros/home/aspx.

[26]A number of additional transformations are made. For GDP, private consumption, investment, and government consumption, we take the first difference of their natural logs. The inflation rates are the log first differences of the underlying indexes. Copper output, foreign output, the real exchange rate, and oil and copper prices are transformed into natural logs. The interest rate series are measured as logs of gross quarterly effective rates. We stationarize copper output and foreign output by fitting linear trends over the sample period by least squares, while the remaining variables are demeaned using their average sample values, except for the real copper price and its persistent component, of which we subtract the estimated unconditional mean from section 3.2.

Table 4. Calibrated Parameters

Parameter	Value	Description
Steady-State Values		
g_Y	2%	Balanced Growth Path (Net Rate, Annual Basis)
$\bar{\pi}$	3%	Steady-State Inflation Target (Net Rate, Annual Basis)
$\bar{\pi}^*$	3%	Steady-State Foreign Inflation Rate (Net Rate, Annual Basis)
$\bar{r}^*$	4.5%	Steady-State Foreign Interest Rate (Net Rate, Annual Basis)
$\bar{\Theta}$	1.3%	Steady-State Country Premium (Net Rate, Annual Basis)
$\bar{s}_B$	0%	Steady-State Structural Fiscal Balance Target
$\bar{g}$	0.060	Steady-State Government Consumption (Stationary Level)
$\bar{a}_S$	0.081	Steady-State Productivity in Commodity Sector
$\bar{y}*$	1	Steady-State Foreign Demand (Stationary Level)
$\bar{p}_O, \bar{p}_S$	1	Steady-State Real Prices of Oil and Commodity Good (Dom. Currency)
Households		
λ	0.5	Share of Non-Ricardian Households
β	0.998	Subjective Discount Factor (Quarterly Basis)
σ_L	1	Inverse Frisch Elasticity of Labor Supply
ψ	20.145	Disutility of Labor Parameter
α_o	0.03	Share of Oil Consumption in Total Consumption
α_c	0.97	Share of Core Consumption in Total Consumption
γ_c	0.79	Share of Domestic Goods in Core Consumption
Non-commodity Sectors		
δ	0.01	Depreciation Rate (Quarterly Basis), Sector *H*
α_H	0.99	Share of Non-oil Inputs in Production, Sector *H*
η_H	0.66	Share of Labor in Cobb-Douglas Value Added, Sector *H*

(*continued*)

Table 4. (Continued)

Parameter	Value	Description
ε_L	11	Elast. of Substitution among Labor Varieties, Sector H
$\varepsilon_L, \varepsilon_H$	11	Elast. of Substitution among Domestic Varieties, Sector H
ε_F	11	Elast. of Substitution among Imported Varieties, Sector F
γ_I	0.66	Share of Domestic Goods in Investment, Sector H
Commodity Sector		
χ	0.33	Government Ownership of Assets, Sector S
τ_S	0.35	Tas Rate on Foreign Profits, Sector S
κ_S	0.009	Fixed Cost of Production, Sector S
δ_S	0.034	Depreciation Rate (Quarterly Basis), Sector S
$1-\eta_S$	0.325	Capital Elasticity of Production, Sector S
γ_{IS}	0.57	Share of Domestic Goods in Investment, Sector S
n	6	Periods of Time to Build (Quarters), Sector S
ϕ_j	0.167	Financial Profile of Investment Projects, Sector S
Foreign Economy		
ζ^*	0.101	Import Share of Foreign Economy
Exogenous Processes		
ρ_y^*, σ_y^*	0.893, 0.0054	AR(1) Coef. and Innov. S.D., Foreign Demand Shock
ρ_i^*, σ_i^*	0.968, 0.0010	AR(1) Coef. and Innov. S.D., Foreign Interest Rate Shock
ρ_π^*, σ_π^*	0.440, 0.0261	AR(1) Coef. and Innov. S.D., Foreign Inflation Shock
ρ_{pO}^*, σ_{pO}^*	0.898, 0.1424	AR(1) Coef. and Innov. S.D., International Oil Price Shock
ρ, σ_u	0.994, 0.0831	AR(1) Coef. and Innov. S.D., Persistent Commodity Price Shock
σ_a	0.4301	Innov. S.D., Transitory Commodity Price Shock

sample period, given an average labor force growth rate of around 2 percent. The steady-state inflation target is set to the central bank's CPI inflation target of 3 percent. The average foreign inflation rate is set to the same value, so that the nominal exchange rate exhibits no trend appreciation or depreciation. The household's subjective discount factor (β) is approximately equal to 0.998 to match a steady-state real interest rate of 2.8 percent, in line with existing estimates of the neutral real interest rate for Chile; see Fuentes and Gredig (2008). The steady-state nominal foreign interest rate is set to 4.5 percent, also in line with Fuentes and Gredig (2008). The average country premium (Θ) consistent with these choices is around 130 basis points. The fiscal balance target ratio ($\bar{s}_B$) is set to 0, consistent with the fact that the fiscal rule started out with a structural fiscal surplus of 1 percent of GDP in 2001, whereas the Chilean government has later targeted a deficit of 1 percent. The steady-state values of a number of exogenous variables (foreign output and the relative international prices of oil and copper) are normalized to one. The value of steady-state government consumption is chosen to match a nominal government consumption-to-GDP ratio of 12 percent, and the steady-state value of productivity in the commodity sector is chosen to match an average share of mining-based GDP in total GDP of around 13 percent over the sample period. The foreign import share (ζ^*) is chosen to match a current account-to-GDP ratio of about –1.8 percent, as in Medina and Soto (2007) and Medina, Munro, and Soto (2008).

Further, the disutility of labor parameter (ψ) is chosen to normalize the steady-state real wage conveniently to one. The share of non-Ricardian households is calibrated to 0.5, following Medina and Soto (2007) and Céspedes, Fornero, and Galí (2011). The share of oil in the household's consumption basket is chosen to match its weight in the Chilean CPI, i.e., 3 percent. The shares of domestic goods in core consumption and investment in the domestic goods sector are set to 79 percent and 66 percent, respectively, based on input-output tables from national accounts since 2008. The labor supply elasticity (σ_L) is set to one and the quarterly depreciation rate of capital in the domestic goods sector (δ) is set to 1 percent, in line with the literature (see, for instance, Adolfson et al. 2008). Following Medina and Soto (2007), the labor share in the Cobb-Douglas production function of value added in the domestic goods sector (η_H) is set to

0.66, the share of non-oil inputs in production (α_H) is set to 0.99, and the elasticities of substitution among labor varieties, domestic varieties, and imported varieties (ε_L, ε_H, and ε_F) are set to 11. The latter imply steady-state wage and price markups of 10 percent.

Turning to the commodity sector, we calibrate the share of government ownership in this sector (χ) to 0.33, consistent with the average share of production of the state-owned copper mining company (Codelco) relative to total copper production since 2001. The tax rate on foreign profits (τ_S) is set to 0.35, which is the flat-rate tax on foreign companies in Chile. The fixed costs in production parameter (κ_S) is chosen to match a labor share in total value added of about 13 percent, according to input-output tables since 2008. The elasticity of production with respect to capital ($1 - \eta_S$) is set to 0.32, in order to match a share of physical capital to quarterly output in the commodity sector of around 8. The quarterly depreciation rate of capital in the commodity sector (δ) is approximately 3 percent, chosen to match an average investment-to-output ratio in Chile's mining sector of about 4 percent from 2001 through 2014. The home bias in investment in the commodity sector is set to 0.57, consistent with available data on the share of construction in total investment in the mining sector. The horizon of time to build is set to $n =$ six quarters, consistent with the average duration of investment projects by private mining companies according to data from a regular survey of the Chilean Corporation of Capital Goods (CBC). Following Kydland and Prescott (1982), the financing profile of projects (ϕ_j) is then set to $1/n$.

We also fix the parameters of the observable exogenous processes. Apart from the estimated parameters of the copper price process from section 3.2, this concerns the processes for foreign demand, the foreign interest rate, foreign inflation, and the real oil price. The parameters of these processes are estimated by constrained maximum likelihood.

Finally, we apply the strategy of adding observation errors on all observed domestic endogenous variables except the interest rate, as in, e.g., Adolfson et al. (2008) and Christiano, Trabandt, and Walentin (2011) for two reasons. First, without these our model would feature a stochastic singularity since we have sixteen observed variables but only fourteen structural shocks (and we assume no additional ad hoc shocks such as price or wage markup shocks).

Second, we know that the data series used are not perfectly measured and are at best only approximations of the "true" series (e.g., due to imperfect seasonal adjustment). We calibrate the variance of the observation errors to 10 percent of the sample variance of the respective variable. Given this value, the measurement errors capture some of the high-frequency noise in the data, while the model still explains 90 percent of the unconditional variances.

We then specify independent univariate prior distributions for the estimated parameters as documented in table 5. The types of the priors are chosen according to the domain on which the individual parameters are defined, while the prior means and standard deviations are specified to reflect our beliefs on plausible regions for the respective parameters. To obtain the joint posterior distribution, we generate a chain of 750,000 draws using the Metropolis-Hastings algorithm and drop the first 250,000 draws for convergence.[27] Based on the remaining draws, the posterior mean and probability intervals are computed. The estimates are presented in table 5. For space reasons, we do not discuss those estimates in more detail.

5. Current Account Dynamics under Learning

In this section we present selected results from the estimated DSGE model. We first discuss the effects of commodity price shocks on the current account, highlighting the role of imperfect information and learning as well as the role of investment in the commodity sector. We then present the historical decompositions of investment and the current account in Chile during 2001:Q3–2015:Q4, emphasizing the impact of the (perceived) transitory and persistent commodity price shocks in comparison with other shocks to understand the evolution of those variables.[28]

5.1 The Impact of Imperfect Information and Learning

Figure 9 shows the simulated responses of investment and the current account balance (expressed as percentages of nominal GDP)

[27]The mode of the posterior kernel that is used to initialize the Metropolis-Hastings algorithm is obtained through the numerical optimization routine developed by Sims (1999).

[28]All results are computed using the posterior mean obtained from the Metropolis-Hastings algorithm.

Table 5. Prior and Posterior Distributions

Parameter	Description	Prior			Posterior	
		Dist.	Mean	S.D.	mean	90% HPDI
Households						
h	Habit Formation	B	0.7	0.1	0.876	[0.824, 0.925]
ω_C	Elast. of Subst. Oil and Core Cons.	IG	1	Inf	0.341	[0.211, 0.472]
η_C	Elast. of Subst. H and F Core Cons.	IG	1	Inf	0.904	[0.253, 1.600]
Wages						
ϕ_L	Calvo Probability Wages	B	0.75	0.1	0.992	[0.987, 0.997]
ξ_L	Indexation Wages	B	0.5	0.2	0.901	[0.819, 0.987]
Prices						
ϕ_{H_D}	Calvo Probability Domestic Prices	B	0.75	0.1	0.365	[0.242, 0.489]
ϕ_{H_F}	Calvo Probability Export Prices	B	0.75	0.1	0.926	[0.872, 0.979]
ϕ_F	Calvo Probability Import Prices	B	0.75	0.1	0.586	[0.401, 0.808]
ξ_{H_D}	Indexation Domestic Prices	B	0.5	0.2	0.336	[0.070, 0.585]
ξ_{H_F}	Indexation Export Prices	B	0.5	0.2	0.413	[0.087, 0.723]
ξ_F	Indexation Import Prices	B	0.5	0.2	0.326	[0.075, 0.564]
Production						
ω_H	Elast. of Subst. Oil and Other Inputs	IG	1	Inf	8.038	[4.503, 11.804]
Investment						
η_I	Elast. of Subst. H and F Inv., Non-S	IG	1	Inf	1.016	[0.230, 1.983]
η_{I_S}	Elast. of Subst. H and F Inv., S	IG	1	Inf	3.052	[2.206, 3.906]
μ_S	Inv. Adjustment Cost Elast., Non-S	G	2	0.5	1.853	[1.170, 2.526]
μ_{I_S}	Inv. Adjustment Cost Elast., S	G	3	0.5	3.052	[2.206, 3.906]
For. Economy						
η_F	Price Elast. Foreign Demand	IG	1	Inf	0.261	[0.185, 0.335]
ϱ	Country Premium Debt Elast.	IG	0.01	Inf	0.004	[0.002, 0.005]

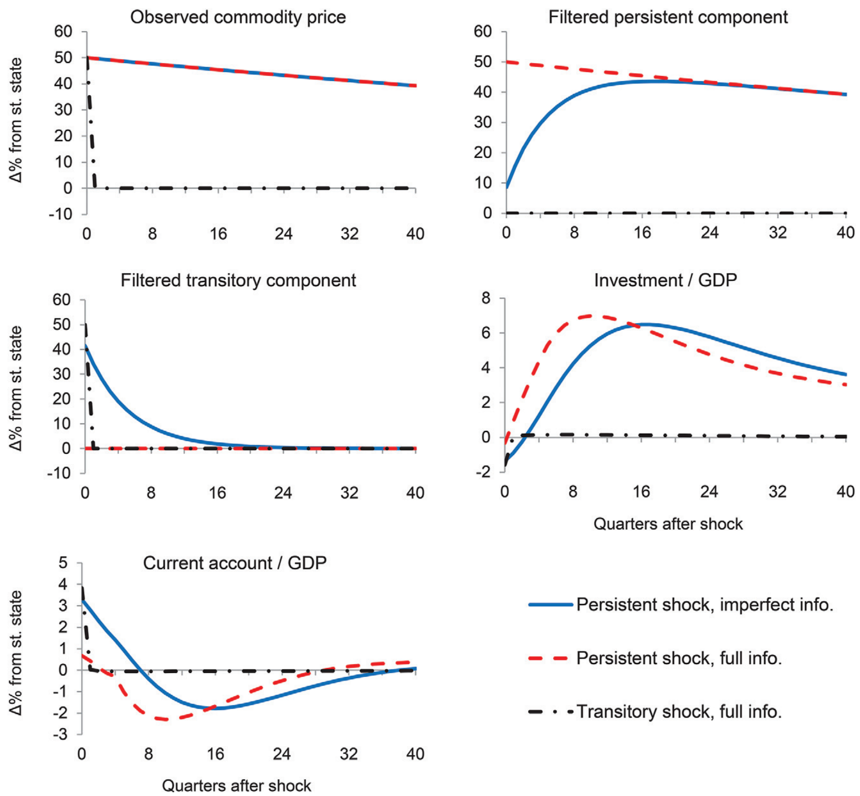
(continued)

Table 5. (Continued)

Parameter	Description	Prior			Posterior	
		Dist.	**Mean**	**S.D.**	**mean**	**90% HPDI**
Mon. Policy						
ψ_r	Interest Rate Smoothing	B	0.75	0.1	0.857	[0.827, 0.888]
ψ_Y	Interest Rate Feedback GDP Growth	N	0.125	0.05	0.056	[-0.019, 0.130]
ψ_π	Interest Rate Feedback Inflation	N	1.5	0.1	1.679	[1.553, 1.814]
$\psi_{\pi Z}$	Feedback Weight Core Inflation	B	0.5	0.2	0.563	[0.303, 0.818]
AR(1) Coef.						
ρ_{a_H}	Neutral Technology Shock	B	0.75	0.1	0.919	[0.875, 0.964]
ρ_{ζ_T}	Productivity Growth Shock	B	0.75	0.1	0.910	[0.866, 0.956]
ρ_{ζ_C}	Preference Shock	B	0.75	0.1	0.725	[0.597, 0.859]
ρ_{ζ_I}	Inv.-Specific Technology Shock	B	0.75	0.1	0.683	[0.557, 0.808]
ρ_{ζ_G}	Fiscal Policy Shock	B	0.75	0.1	0.881	[0.820, 0.944]
$\rho_{\zeta\Theta}$	Country Premium Shock	B	0.75	0.1	0.953	[0.919, 0.988]
ρ_{a_S}	Commodity-Specific Technology Shock	B	0.75	0.1	0.992	[0.986, 0.998]
Innov. S.D.						
u_{a_H}	Neutral Technology Shock	IG	0.005	Inf	0.0060	[0.0047, 0.0074]
u_{ζ_T}	Productivity Growth Shock	IG	0.005	Inf	0.0055	[0.0032, 0.0078]
u_{ζ_C}	Preference Shock	IG	0.005	Inf	0.0440	[0.0248, 0.0618]
u_{ζ_I}	Inv.-Specific Technology Shock	IG	0.005	Inf	0.0469	[0.0295, 0.0635]
u_{ζ_G}	Fiscal Policy Shock	IG	0.005	Inf	0.0382	[0.0320, 0.0443]
u_{ζ_m}	Monetary Policy Shock	IG	0.005	Inf	0.0017	[0.0014, 0.0020]
u_{ζ_Θ}	Country Premium Shock	IG	0.005	Inf	0.0009	[0.0007, 0.0010]
u_{a_S}	Commodity-Specific Technology Shock	IG	0.005	Inf	0.0353	[0.0299, 0.0404]

Notes: The prior distributions are beta distribution (B) on the open interval $(0, 1)$, inverse gamma distribution (IG) on $\mathbb{R}^+$, gamma distribution (G) on $\mathbb{R}_0^+$, and normal distribution (N) on $\mathbb{R}$. The posterior distribution is computed from a Metropolis-Hastings chain of 750,000 draws with an average acceptance rate of approximately 25 percent, dropping the first 250,000 draws for convergence. The reported interval is the 90 percent highest posterior density interval (HPDI).

Figure 9. Impulse Responses under Imperfect versus Full Information



Notes: This figure shows simulated impulse responses of selected variables to persistent (u_t) and transitory (a_t) commodity price shocks of 50 percent based on the estimated DSGE model. The solid lines correspond to a persistent shock in the baseline model with imperfect information on the persistence of the commodity price and with learning. The dashed lines correspond to a persistent shock under full information. The dash-dotted lines correspond to a transitory shock under full information. The results are based on the posterior mean of the parameters. The commodity price and its components are expressed as relative percentage deviations from steady state, while investment and the current account balance are expressed as percentages of nominal GDP in absolute deviations from steady state.

for three different cases: (i) a persistent commodity price shock under imperfect information with the learning mechanism active (i.e., $K = 0.17$ with $\rho = 0.994$, as estimated); (ii) the hypothetical case of a persistent shock under full information without learning (i.e., $K = \rho = 0.994$), and (iii) the hypothetical case of a transitory (iid) shock under full information. The aim of this exercise is to analyze the impact of imperfect information and learning on the dynamics of domestic investment and the external savings balance. In each case, the size of the shock is normalized to 50 percent, which roughly corresponds to the observed increase of the real price of copper towards the mid-2000s relative to its mean (see figure 7).

The first three plots of figure 9 show the responses of the actual commodity price and its filtered persistent and transitory components that reflect the agents' changing beliefs. Under imperfect information, when faced with the higher commodity price, agents believe at first that the price increases are mostly transitory because the transitory component is more volatile than the persistent one. However, the agents revise their beliefs over time as they observe that the effective price remains high, attributing a larger weight to the persistent component. In this way, the short-term responses of investment and the current account (and other variables) to the persistent shock fall in between the corresponding responses to persistent and transitory shocks in the full-information case. In that case, investment almost does not react to the transitory shock, but it increases under the persistent shock to more than 6 percent of GDP. But under imperfect information, investment increases more gradually given the agents' changing perception of the persistence of the shock. Regarding the current account, under imperfect information the shock generates an initial surplus of around 3 percent of GDP, followed by a slowly declining and eventually negative balance reaching approximately −2 percent of GDP. The reason for the current account reversal is the uncertainty about the duration of the shock, as the comparison with the full-information case shows: most of the income gains are saved as agents believe at first that the shock may be transitory, but as they realize that the shock is more persistent than thought, a larger fraction of domestic investment is financed through a current account deficit.

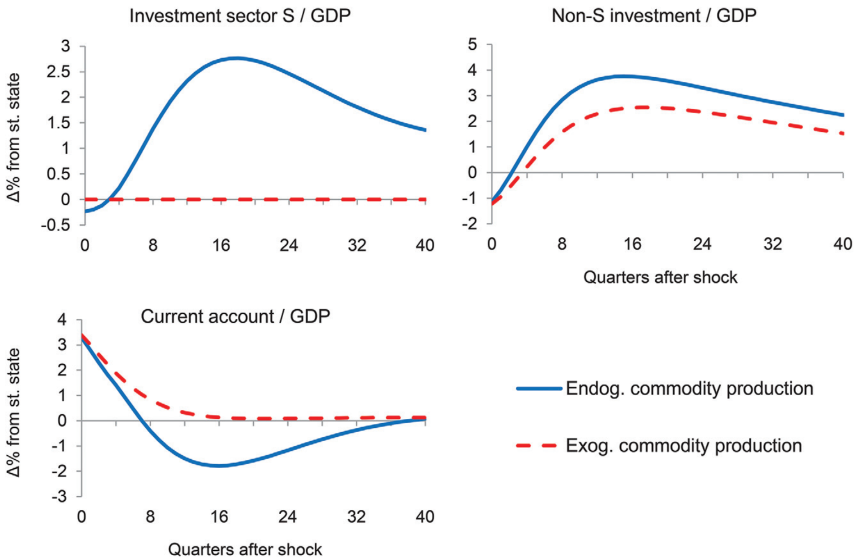
Note that under full information, a current account deficit would materialize almost instantaneously, while a transitory price increase would mainly be saved according to the estimated DSGE model. The model's dynamics are thereby consistent with standard theory on saving-investment behavior in small open economies, as in Obstfeld and Rogoff (1995), although the nominal and real rigidities present in our model add of course some sluggishness. However, the results show that the presence of imperfect information and learning is key to reproduce the empirical observations on current account dynamics in Chile and other commodity-exporting economies discussed in section 2—in particular, the observed reversals.

5.2 The Role of Commodity Investment

Another key factor for the response of the current account to a commodity price shock is the presence of investment in the commodity sector. In particular, the shock generates a stepwise increase in commodity investment with significant spillovers to other sectors. These spillovers and related empirical evidence are discussed in detail in Fornero, Kirchner, and Yany (2015). Here we focus on the link between commodity investment and the current account.

To examine the role of commodity investment, figure 10 shows the responses of sectoral investment and the current account to the persistent commodity price shock in the baseline model with imperfect information, and compares those responses with the model where commodity production is exogenous such that there are no endogenous movements in commodity investment. In the baseline model, the shock has a strong effect on investment in the commodity sector, which increases by almost 3 percent of GDP. Other investment also increases significantly. Note that investment in the commodity sector reacts more slowly compared with other sectors due to the presence of time to build and larger adjustment costs, which makes firms in this sector invest in a more cautious way given the uncertainty on the duration of the shock. Also note that other investment is stimulated by the demand for domestic goods from the commodity sector. Regarding the current account, the endogenous response of commodity investment is the main factor behind the eventual deficit; without commodity investment, the current account would return to a balanced position after the initial surplus. Hence, together with imperfect information and learning on the persistence of commodity price shocks, the presence of commodity investment is critical to generate the current account reversal.

Figure 10. Impulse Responses under Endogenous versus Exogenous Commodity Production

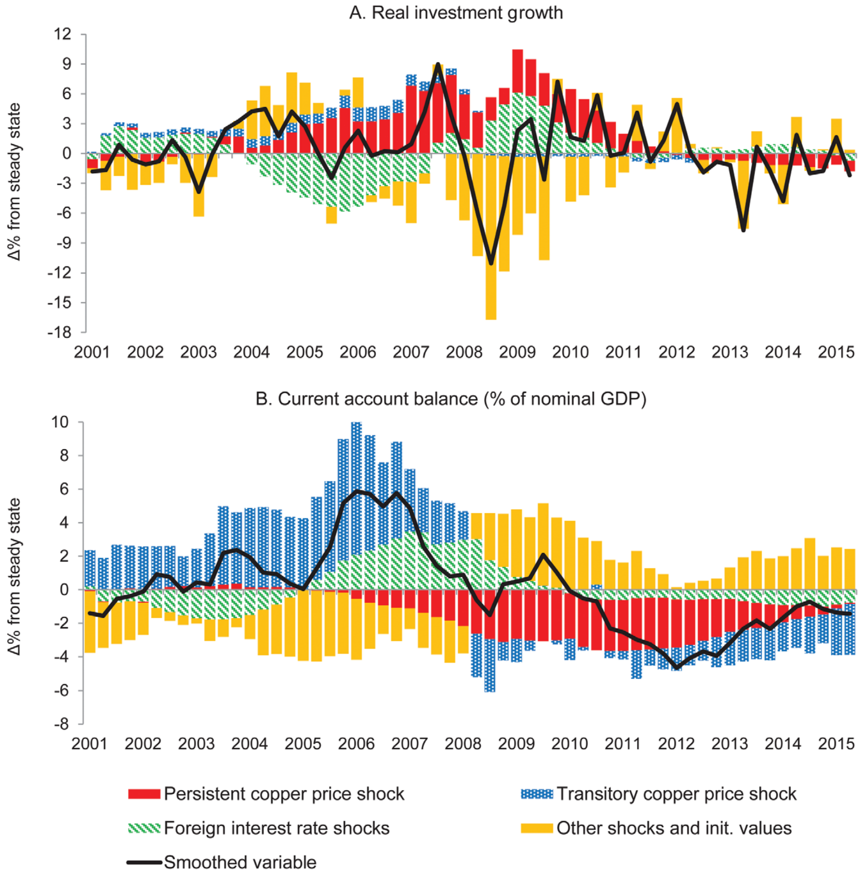


Notes: This figure shows simulated impulse responses of investment in the commodity sector (S) and the non-commodity sector (non-S) and the current account balance to a persistent commodity price shock of 50 percent. The solid lines correspond to the baseline model with endogenous commodity production and imperfect information on the persistence of the commodity price. The dashed lines correspond to the model with exogenous commodity production and no investment in the commodity sector, with the calibrated and estimated parameters as in the baseline model. The results are based on the posterior mean of the parameters. All variables are expressed as percentages of nominal GDP in absolute deviations from steady state.

5.3 Historical Decomposition of Investment and the Current Account

We now assess the consequences of the mechanism described to understand the observed fluctuations of investment and the current account in Chile. Figure 11 shows the historical decompositions of real investment growth and the current account balance, both smoothed and in deviations from their steady-state values. We report the contributions of innovations to the copper price and the foreign interest rate (including shocks to the country spread), and sum up the contributions of any other shocks. The results show that a large

Figure 11. Historical Decomposition of Investment Growth and Current Account in Chile



Notes: This figure shows the estimated contributions to the quarterly growth rate of real investment in percent (chart A) and the current account balance as a percentage of nominal GDP (chart B) from 2001:Q3 through 2015:Q4 of shocks to the filtered persistent and transitory components of the real price of copper, foreign interest rate shocks (including shocks to the country spread), and other shocks plus the initial values of the state variables. The results are based on the posterior mean of the parameters. Both variables are expressed as absolute deviations from steady state.

fraction of the observed investment boom before the 2008–09 crisis and afterwards can be attributed to the inferred persistent increase in the copper price. The current account surplus of the mid-2000s can mostly be associated with the inferred transitory component, whereas the subsequent reversal is mainly explained by the inferred persistent component. Movements in foreign interest rates explain about half of the initial surplus but only a minor fraction of the eventual deficit. Hence, changing perceptions on the persistence of the commodity price shock are the main factor behind the observed evolution of the Chilean current account balance. The facts discussed in section 2 suggest that the same mechanism may be relevant for other commodity exporters, too.

6. Conclusions

This paper has incorporated imperfect information by economic agents on the persistence of commodity price shocks, captured by means of a simple unobserved-components model for commodity prices with persistent and transitory shocks, and an associated learning problem based on the Kalman filter into a small open-economy DSGE model with a commodity sector. In this model, investment in the commodity sector only responds to persistent changes in commodity prices due to time to build and adjustment costs. Then, the prediction that a current account reversal (from surplus to deficit) can materialize in response to a persistent increase in commodity prices relies on both mechanisms at the same time: the first—imperfect information and learning—is responsible for slowly changing perceptions on the persistence of price changes, whereas the second—time to build with adjustment costs—makes investment projects feasible only if the expected price in the near medium term is high.

Based on the estimation of parameters of the model using Chilean quarterly data from 2001:Q3–2015:Q4, we were able to conduct several quantitative experiments. One of the key predictions of the model is that during a lasting increase in commodity prices, agents believe at first that the price increase is mostly temporary but revise their expectations over time as they observe that commodity prices remain high, and thus gradually decrease savings and increase investment especially in the commodity sector. This mechanism is apt to explain several stylized facts, including, in particular, the stepwise current account deterioration in Chile and other commodity-exporting economies. Our results thereby allow for a novel interpretation of current account developments in commodity-exporting countries, in comparison with previous related work which has pointed towards investment-specific shocks, changes in foreign financial conditions, and variations in foreign demand (see Medina, Munro, and Soto 2008).

Our analysis allows us to draw some conclusions for macroeconomic policy in commodity-exporting countries. First and foremost, a gradual current account deterioration in a commodity-exporting country in times of a persistent commodity price boom is not necessarily a sign of emerging macroeconomic imbalances, because it may be a natural byproduct of increasing investment in the commodity sector that enhances the productive capacity of the economy. Further, current account deficits that are primarily due to foreign direct investment (FDI) in the commodity sector may be less worrisome to policymakers than other types of external savings deficits—for instance, due to large portfolio inflows—as FDI is often regarded as a more stable source of funding (see, for instance, Albuquerque 2003 and Goldstein and Razin 2006).[29]

Appendix 1. Derivation of the Signal-to-Noise Ratio in the Model for $p^*_{S,t}$

This appendix describes the derivation of the signal-to-noise ratio in the unobserved-components model for the real international copper price described in section 3.1. For convenience, we repeat this model here:

$$\breve{p}^*_{S,t} = \breve{b}_t + a_t, \qquad a_t \sim N(0, \sigma_a^2),$$
$$\breve{b}_t = \rho \breve{b}_{t-1} + u_t, \qquad u_t \sim N(0, \sigma_u^2), \qquad t = 1, \ldots, T.$$

[29]In addition, several studies suggest that FDI may have positive effects on long-term economic growth through technology adoption by host countries. See Borensztein, De Gregorio, and Lee (1998), Markusen and Venables (1999), Javorcik (2004), Li and Liu (2005), and Haskel, Pereira, and Slaughter (2007).

From the Kalman-filter recursions, the steady-state value of the Kalman gain is as follows:

$$K = \rho\Sigma(\Sigma + \sigma_a^2)^{-1},$$

where Σ, the steady-state value of the conditional variance of the unobserved state, satisfies

$$\Sigma = \rho^2 \left[\Sigma - \frac{\Sigma^2}{\Sigma + \sigma_a^2}\right] + \sigma_u^2. \tag{8}$$

The above equation can be rewritten as follows:

$$\left(\frac{\sigma_u}{\sigma_a}\right)^2 = \frac{\Sigma}{\sigma_a^2} - \frac{\rho^2}{1 + \sigma_a^2/\Sigma} = \frac{(\Sigma/\sigma_a^2)^2 + (\Sigma/\sigma_a^2)(1-\rho^2)}{1 + \Sigma/\sigma_a^2}. \tag{9}$$

Rewriting (8) in terms of the ratio Σ/σ_a^2 yields

$$\frac{\Sigma}{\sigma_a^2} = (\rho/K - 1)^{-1}. \tag{10}$$

By (9) and (10), given values of K and ρ, the signal-to-noise ratio satisfies

$$\begin{aligned}\frac{\sigma_u}{\sigma_a} &= \sqrt{\frac{(\rho/K-1)^{-2} + (\rho/K-1)^{-1}(1-\rho^2)}{1 + (\rho/K-1)^{-1}}}\\ &= \sqrt{\frac{(\rho/K-1)^{-1} + 1 - \rho^2}{\rho/K}}.\end{aligned}$$

Appendix 2. Implementing Learning about Exogenous Shocks

This appendix explains in detail our implementation of imperfect information and learning about exogenous shocks in the DSGE model, following Erceg and Levin (2003). Consider a log-linearized model:

$$x_t = AE_t\{x_{t+1}\} + Bx_{t-1} + Cz_t, \qquad t = 1, \ldots, T, \tag{11}$$

where z_t is the exogenous variable that drives the model. The latter has two components, a transitory component, a_t, and a persistent component, b_t:

$$z_t = a_t + b_t. \tag{12}$$

The persistent component follows an AR(1) process:

$$b_t = \rho b_{t-1} + u_t, \qquad u_t \sim N(0, \sigma_u^2). \tag{13}$$

The transitory component is white noise:

$$a_t \sim N(0, \sigma_a^2). \tag{14}$$

The variables a_t and b_t are not observable. The agents infer those unobserved components in an optimal linear way using the Kalman filter, as follows. Let $\mathcal{Z}^t = \{z_1, z_2, z_3, \ldots, z_t\}$ and define $\widehat{b}_t = E[b_t|\mathcal{Z}^t]$, i.e., the inference regarding the persistent component of z_t upon observing current and past values of z_t. The (steady-state) Kalman filter yields

$$\widehat{b}_t = \rho\widehat{b}_{t-1} + K\rho^{-1}(z_t - \rho\widehat{b}_{t-1}). \tag{15}$$

Notice that $\rho\widehat{b}_{t-1}$ is the forecast of b_t and $\widehat{b}_t$ conditional on the information available at time $t-1$, i.e., $E[b_t|\mathcal{Z}^{t-1}] = E[\widehat{b}_t|\mathcal{Z}^{t-1}] = \rho\widehat{b}_{t-1}$ with forecast error $\widehat{u}_t = \widehat{b}_t - \rho\widehat{b}_{t-1}$. Now define $\widehat{a}_t = E[a_t|\mathcal{Z}^t]$, i.e., the inference of the transitory component of z_t with the information available at time t, which satisfies

$$\widehat{a}_t = z_t - \widehat{b}_t. \tag{16}$$

Notice that $E[\widehat{a}_t|\mathcal{Z}^{t-1}] = E[z_t|\mathcal{Z}^{t-1}] - E[\widehat{b}_t|\mathcal{Z}^{t-1}] = E[b_t|\mathcal{Z}^{t-1}] - \rho\widehat{b}_{t-1} = 0$. Analogously, we have that $E[\widehat{a}_{t+1}|\mathcal{Z}^t] = 0$, such that $\widehat{a}_t$ can be interpreted as an exogenous disturbance with conditional mean equal to zero. The model (11) can therefore be represented as follows:

$$x_t = AE_t\{x_{t+1}\} + Bx_{t-1} + C\widehat{a}_t + C\widehat{b}_t, \tag{17}$$

with

$$\widehat{b}_{t+1} = \rho\widehat{b}_t + \widehat{u}_{t+1}, \tag{18}$$

and with (12)–(16). There is one exogenous variable, z_t, and two "perceived" exogenous variables, $\widehat{a}_t$ and $\widehat{b}_t$. However, (17)–(18) can be solved independently from (12)–(16).

Hence, the following algorithm can be used to simulate the response of the endogenous variables in x_t to shocks to z_t:

(i) Solve (17)–(18) to obtain the decision rules

$$x_t = D\widehat{a}_t + E\widehat{b}_t + Fx_{t-1}. \tag{19}$$

(ii) Generate a sequence for z_t using (12)–(14).

(iii) Generate a sequence for $\widehat{b}_t$ using (15).

(iv) Generate a sequence for $\widehat{a}_t$ using (16).

(v) Evaluate (19) using the sequences for $\widehat{b}_t$ and $\widehat{a}_t$ generated in steps (iii) and (iv).

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